How to decide Functionality of Compositions of Top-Down Tree Transducers

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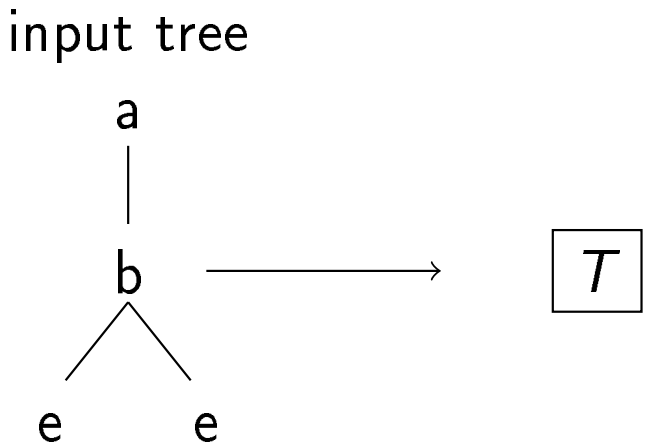

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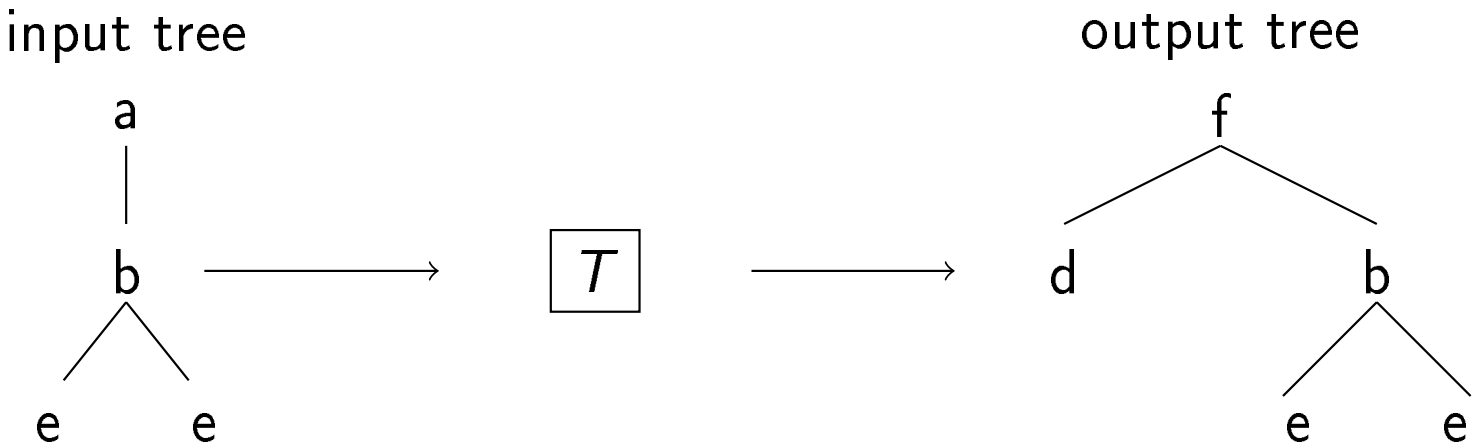
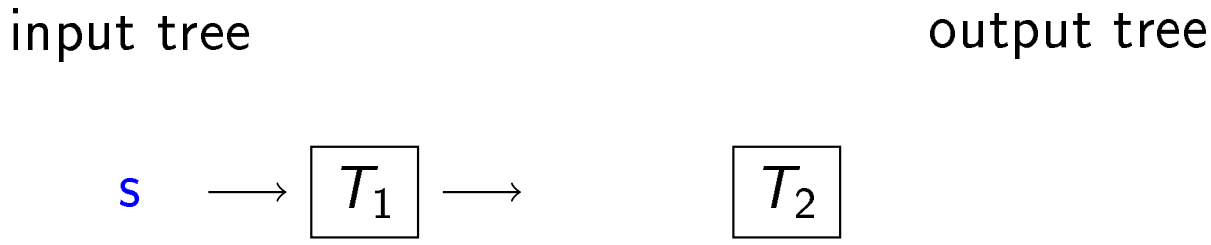


Composition of Transducers



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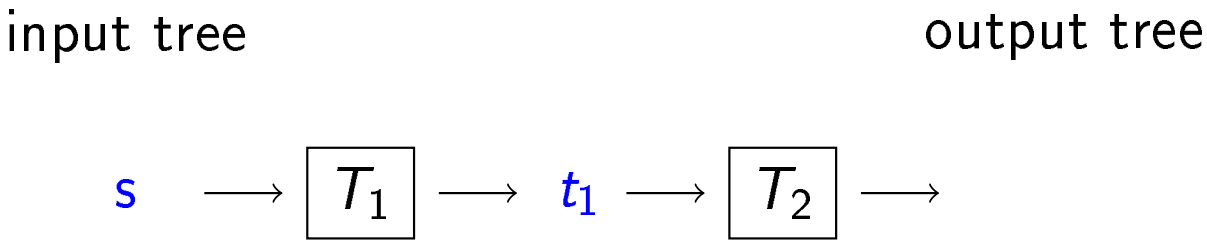
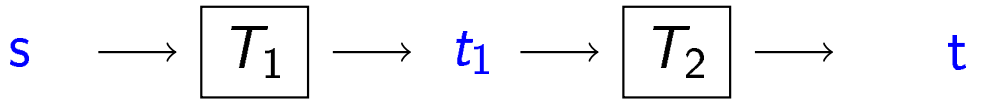


Composition of Transducers

input tree output tree



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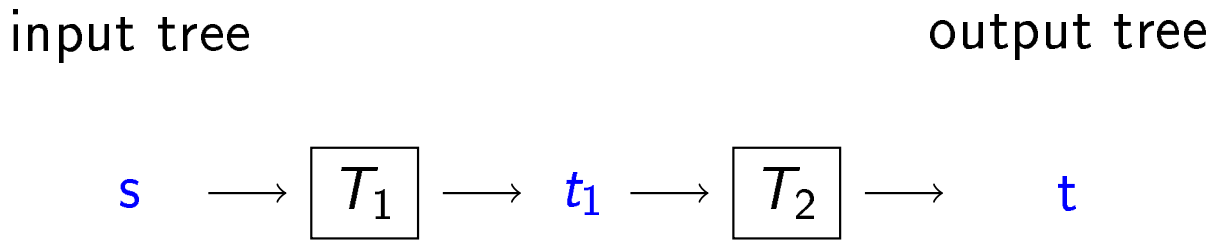

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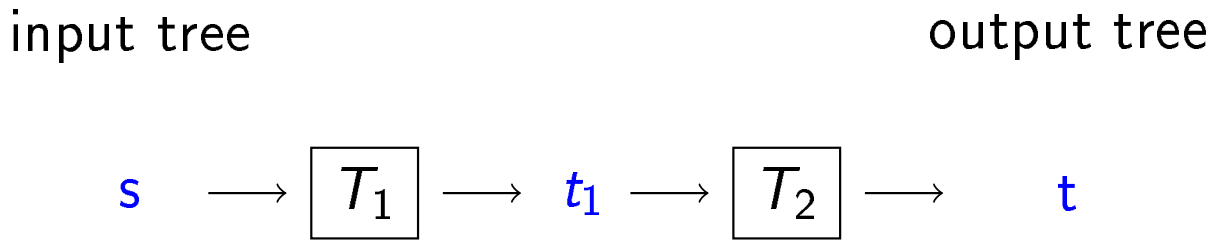

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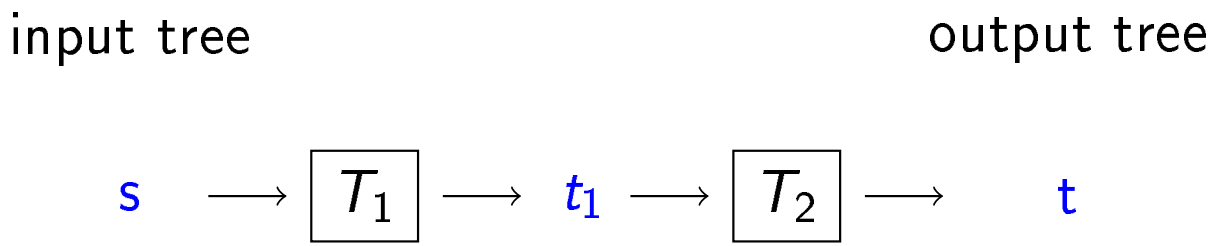

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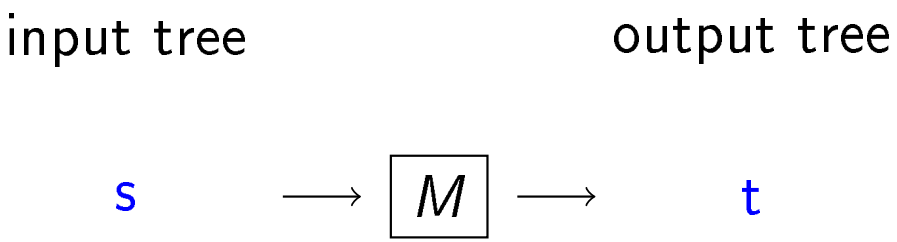

Main Idea

Observation: If the composition of T_1 and T_2 is not functional then M does not exist.

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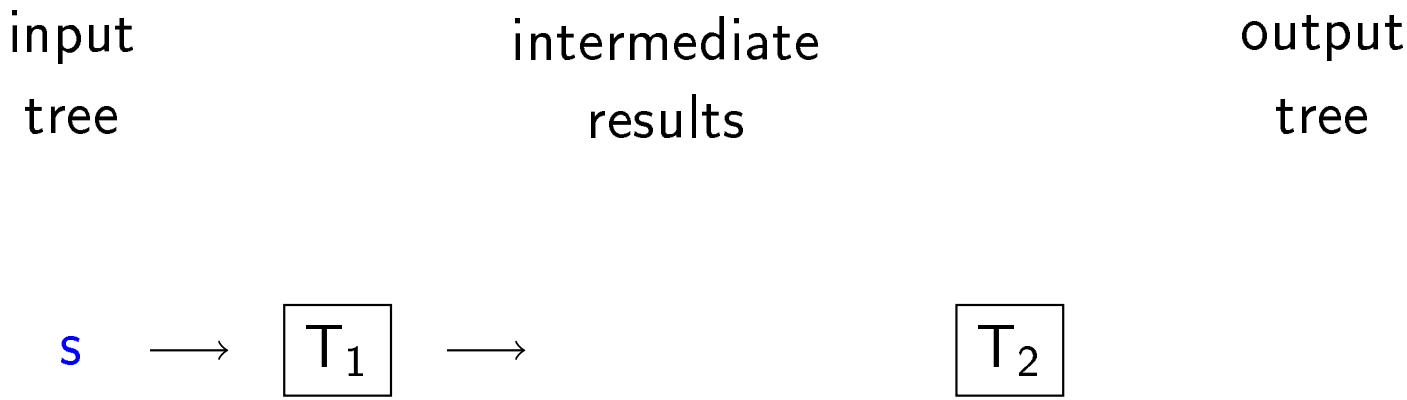

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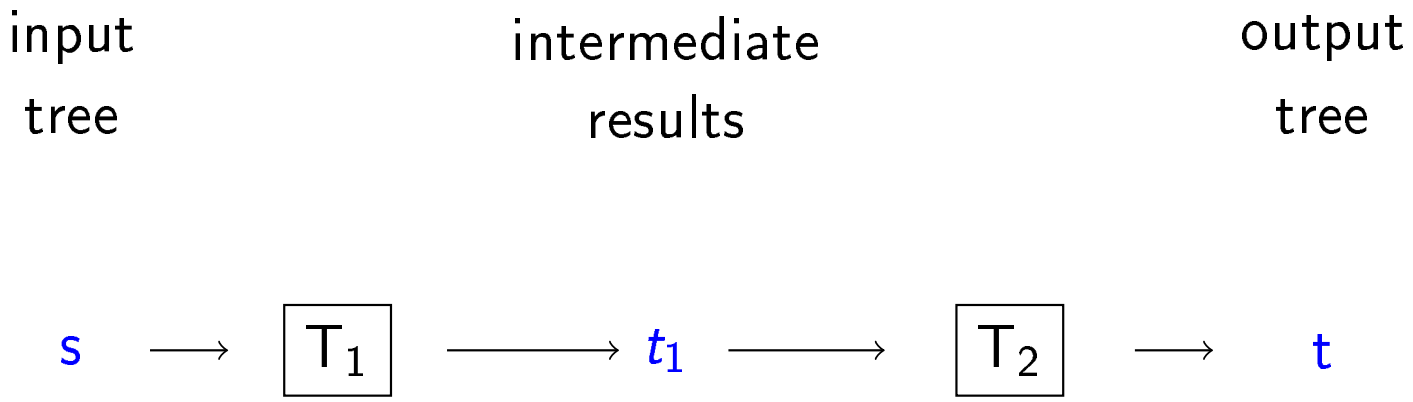

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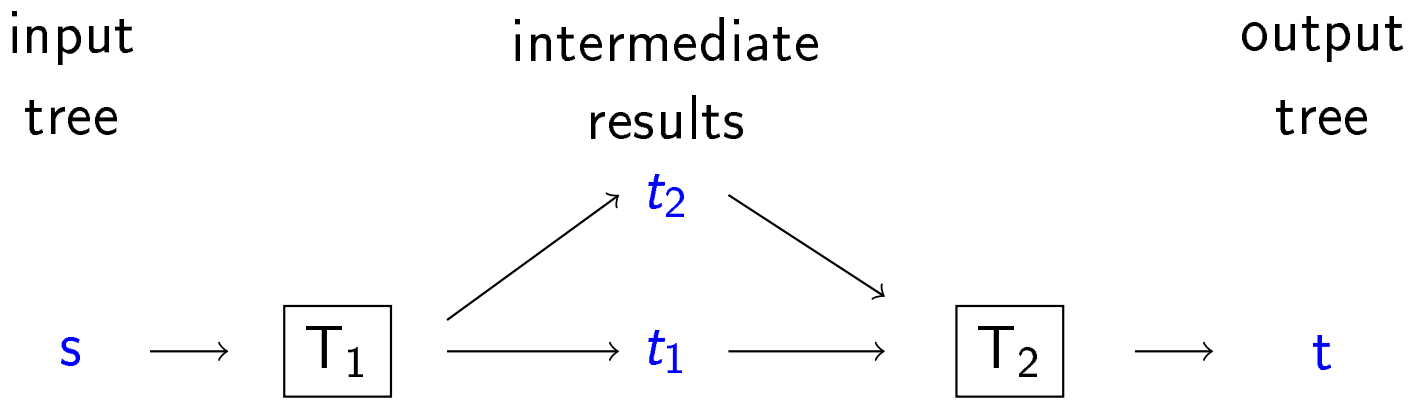

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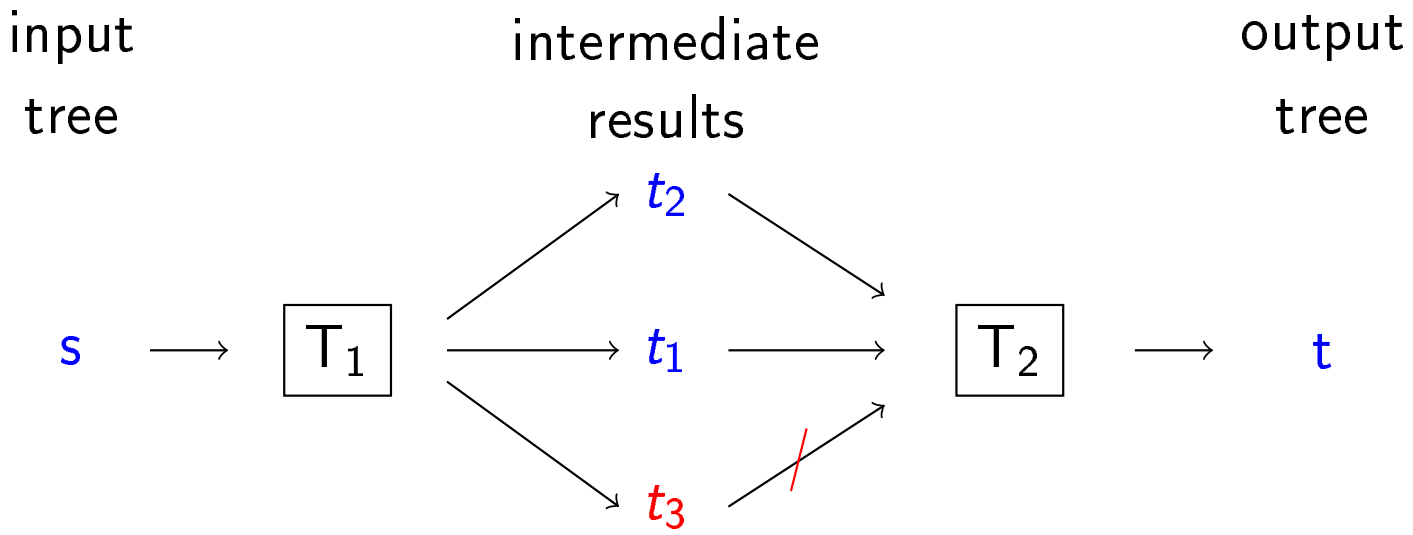

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1. Construct a transducer N with look-ahead such that

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2. Test whether or not N is functional

How to construct N?

Baker has shown that linear, nondeleting transducer are closed under composition

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Main challenges in the construction of N:

1. copying rules and
2. deleting rules

Copying Rules

Transducer T_1:

$$q_1(a(x_1)) \rightarrow b(q_1(x_1)) \qquad q_1(e) \rightarrow e_i \mid i = 1, 2, 3$$

Transducer T_2:

$$\begin{array}{lll} q_2(b(x_1)) \rightarrow f(q_2'(x_1), q_2''(x_1)) & & q_2'(e_j) \rightarrow e \mid j = 1, 2 \\ q_2''(e_3) \rightarrow e' & & q_2''(e_j) \rightarrow e \mid j = 1, 2. \end{array}$$

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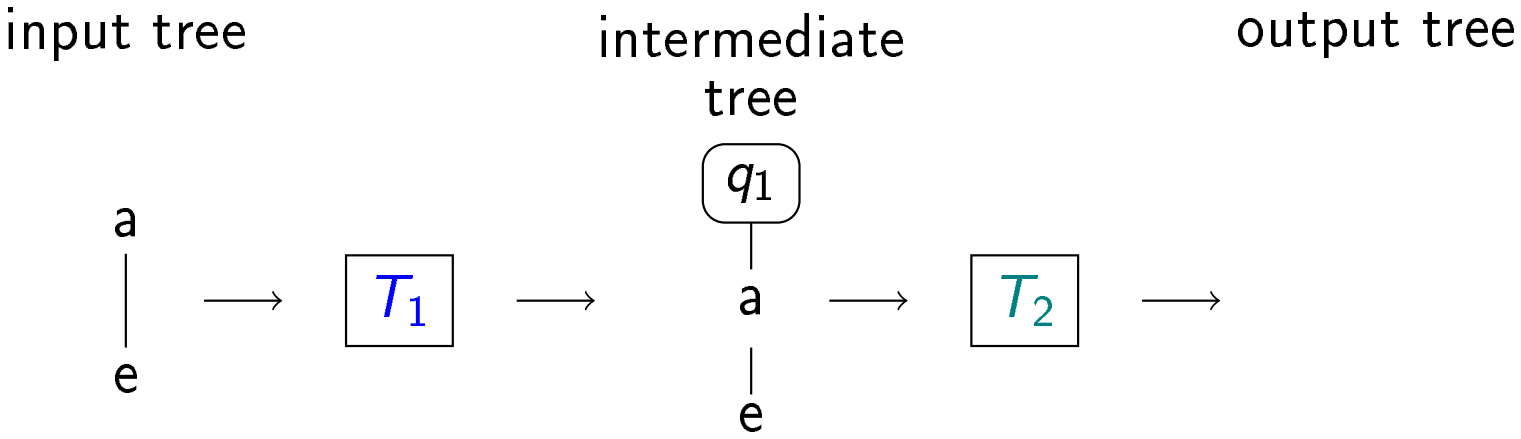

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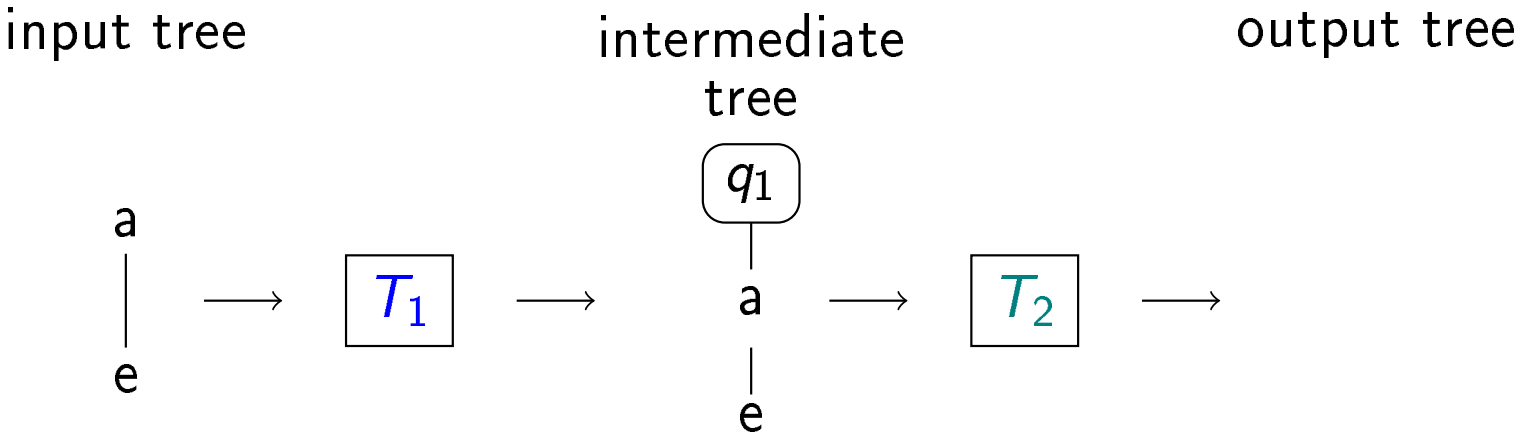

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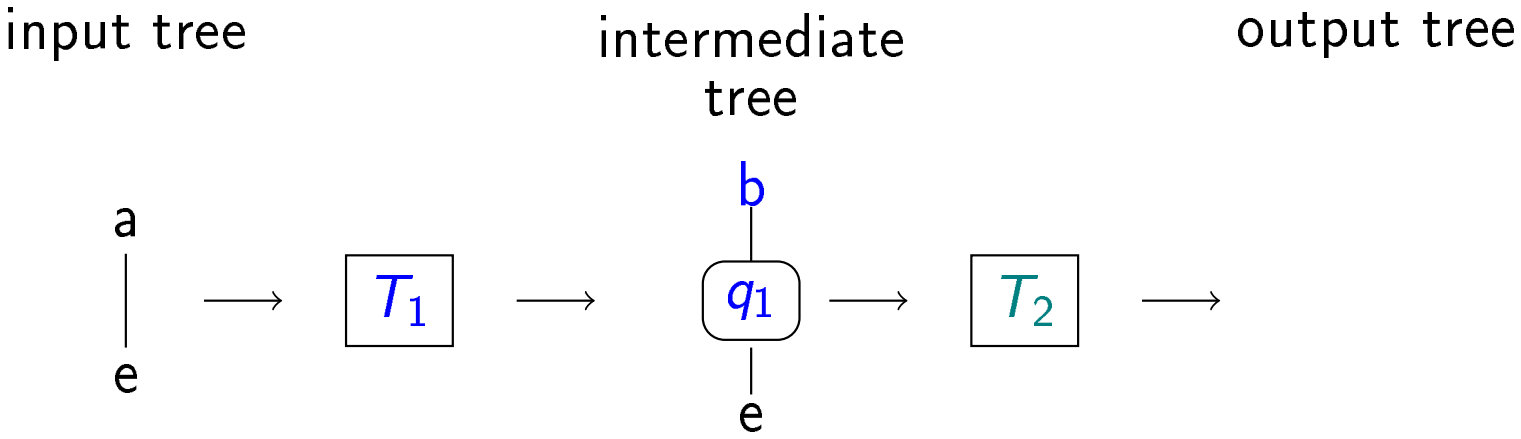

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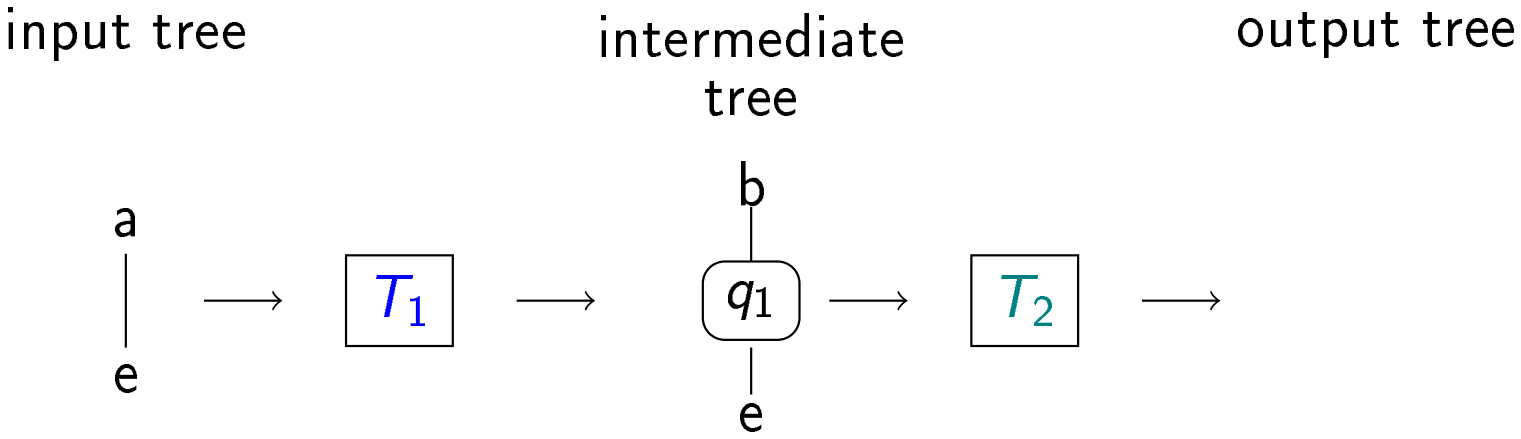

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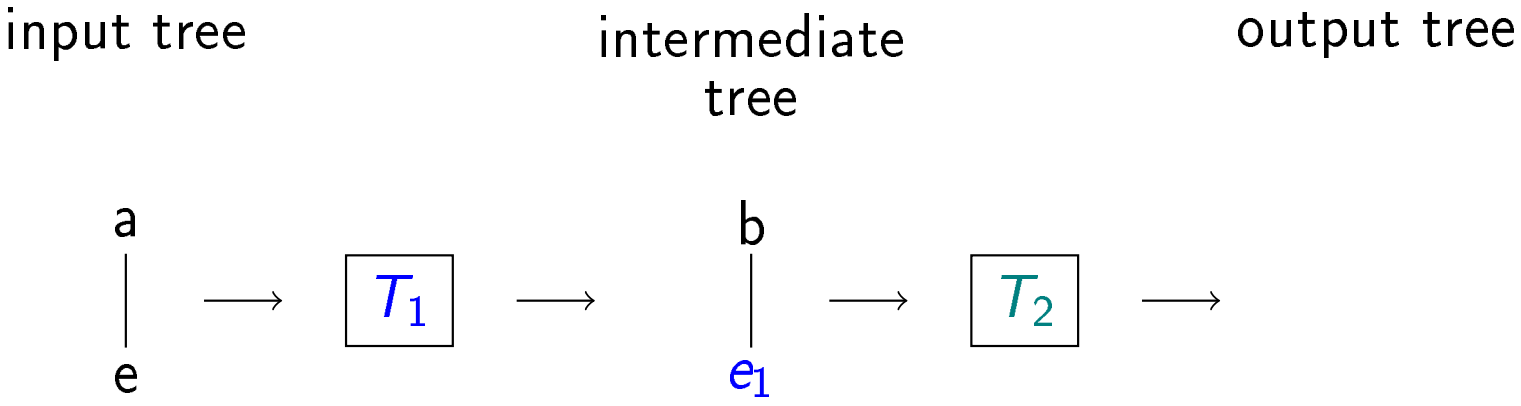

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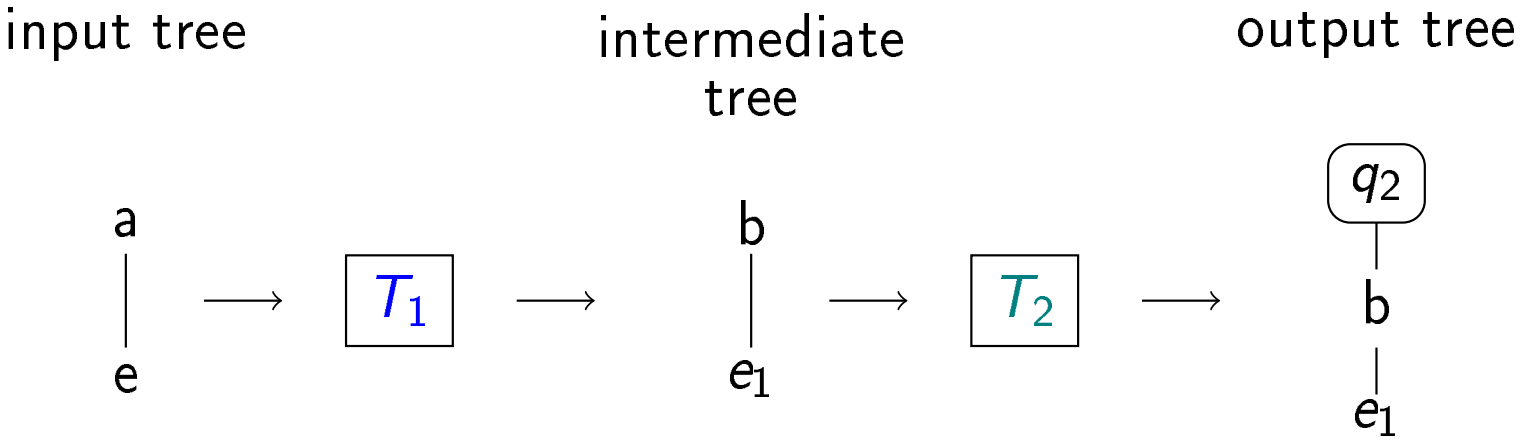

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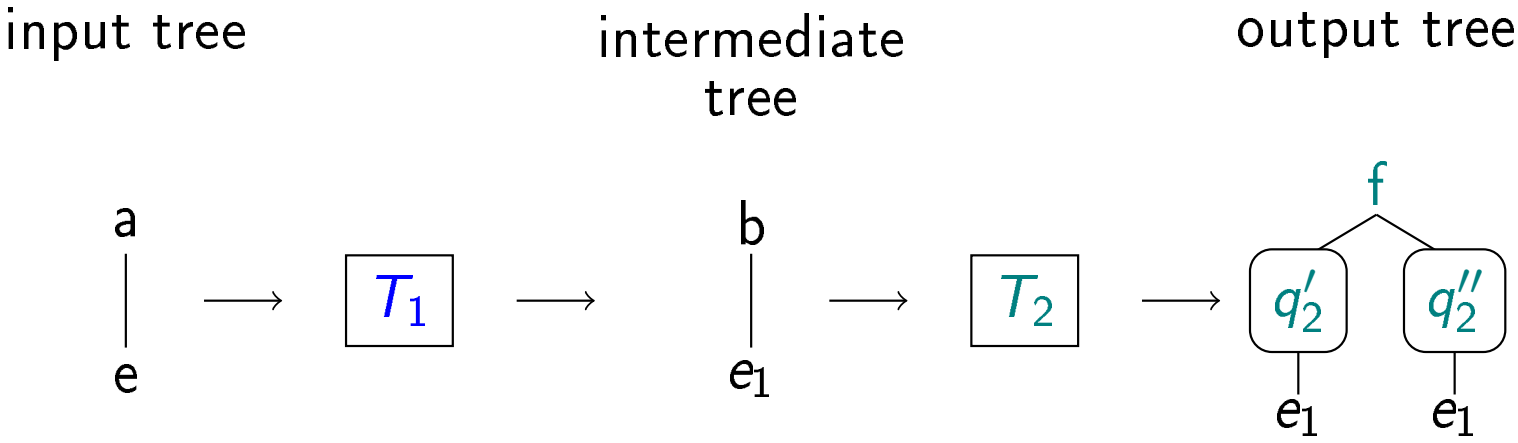

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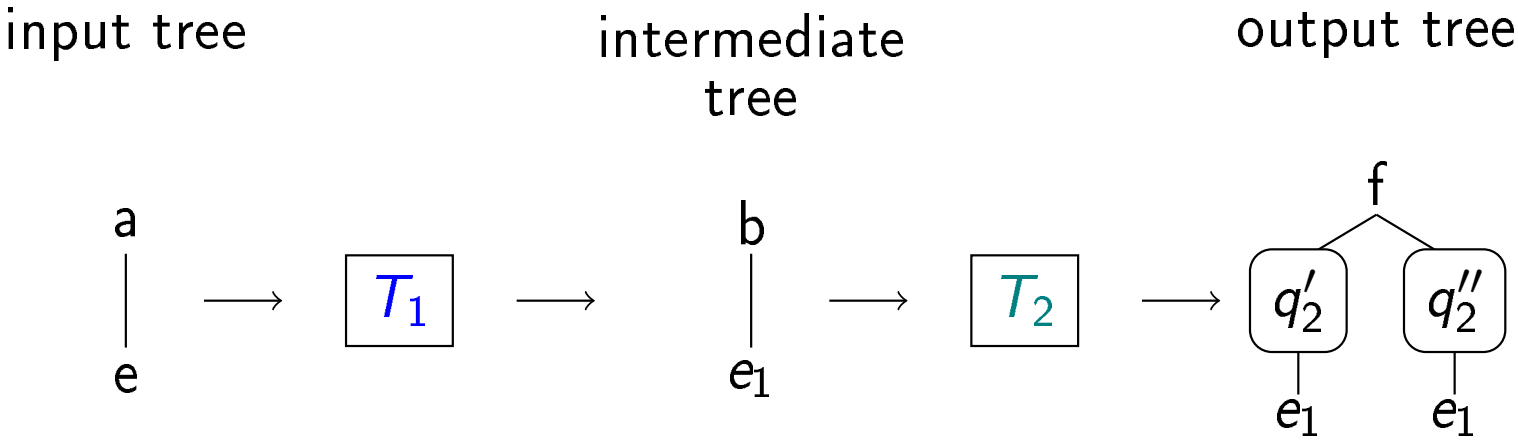

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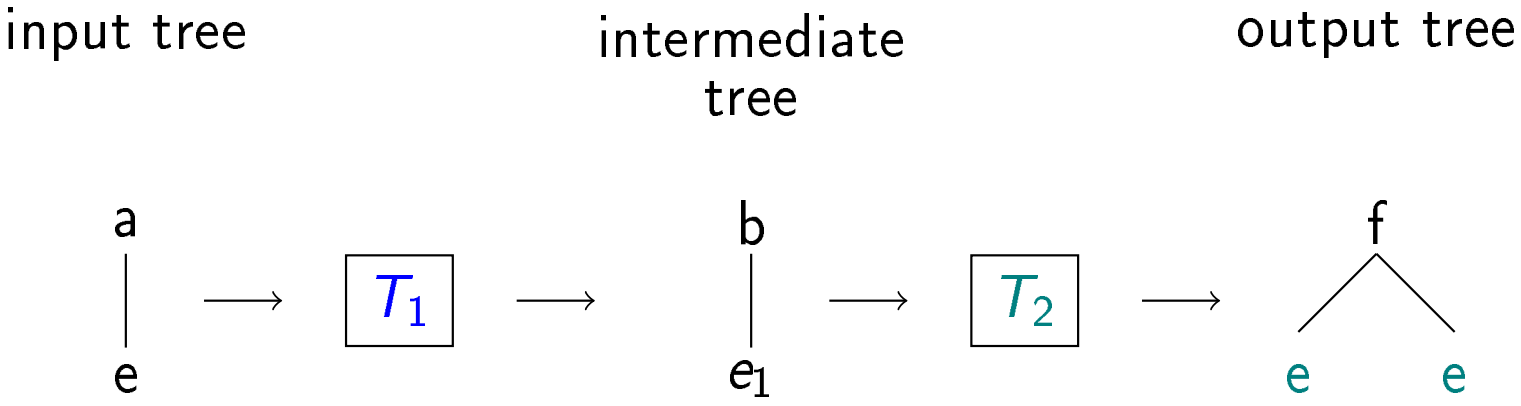

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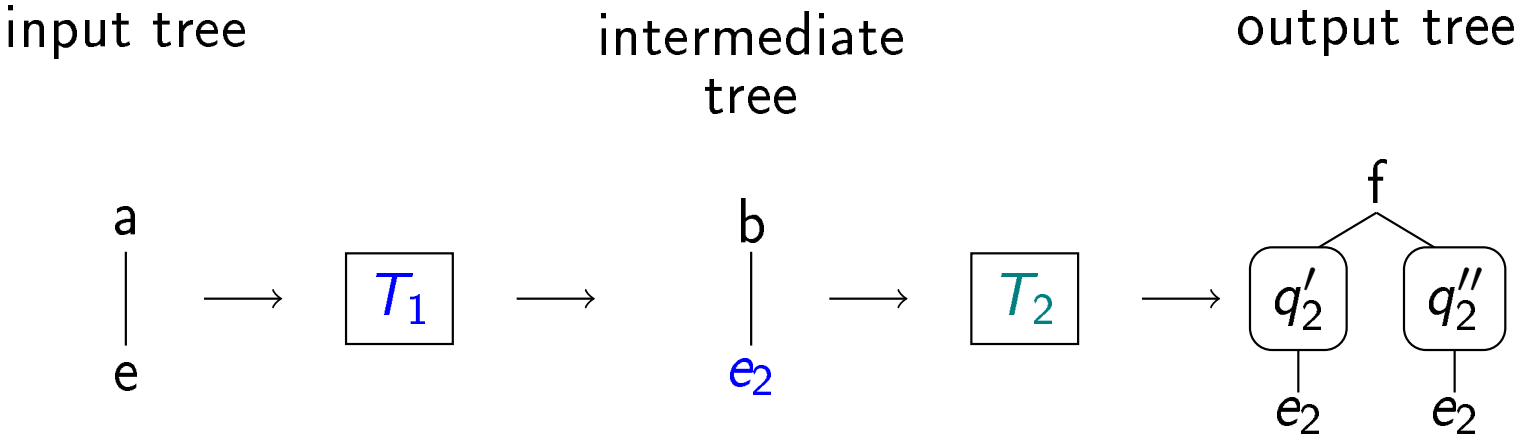

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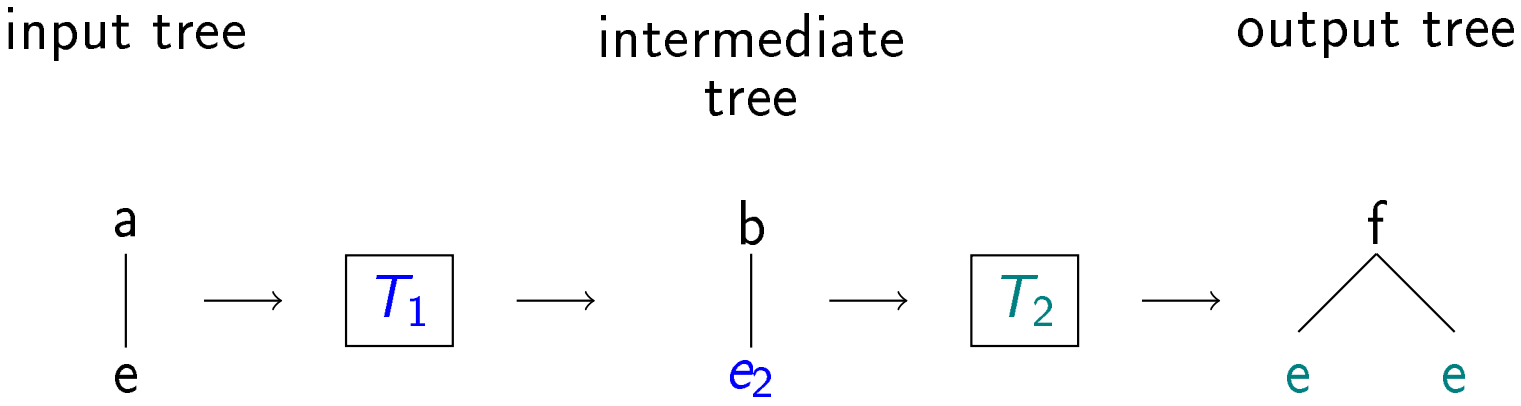

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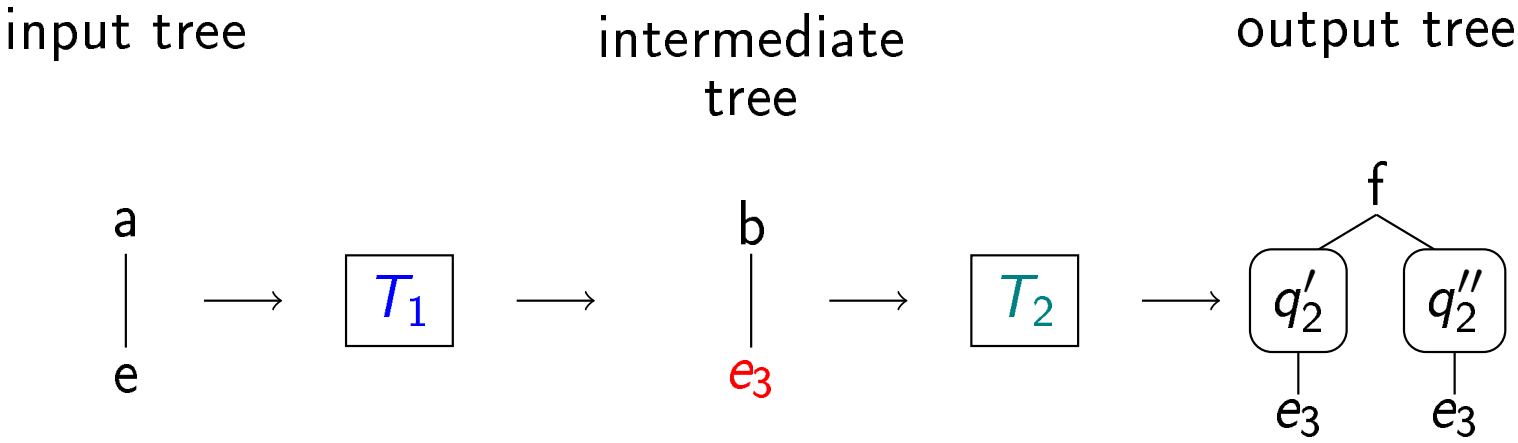

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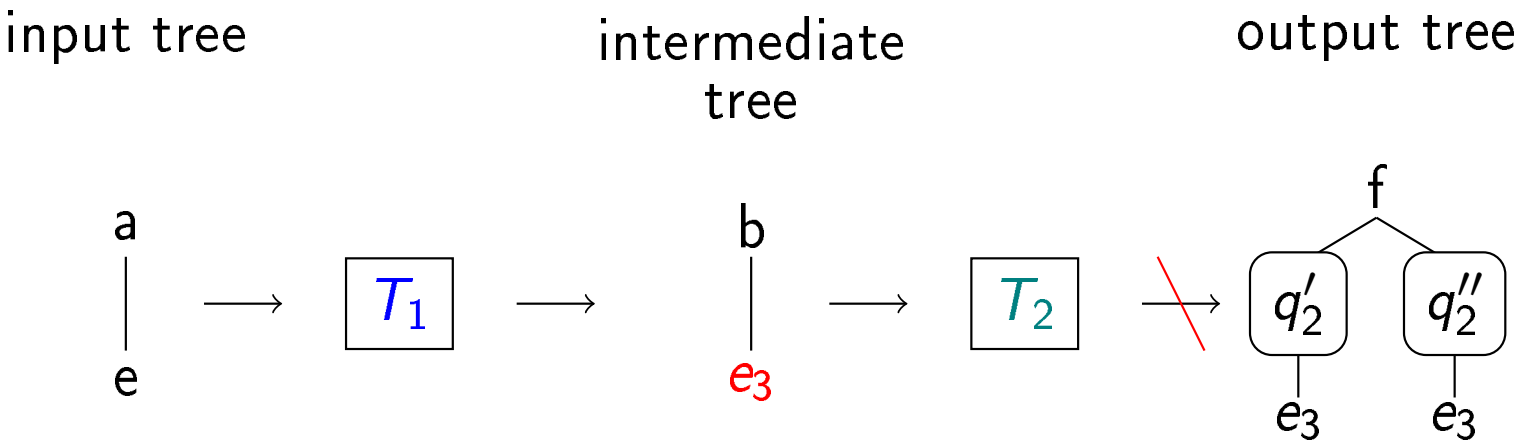

Naive Construction of N

(q_1, q_2)

Naive Construction of N

$$q_1(a(x_1)) \to b(q_1(x_1))$$

$$(q_1, q_2)(a(x_1))$$

Naive Construction of N

$(q_1, q_2)(a(x_1))$

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$q_2(b(q_1(x_1)))$

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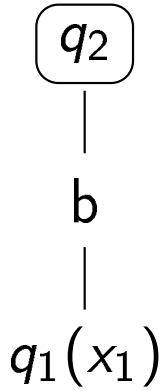

$q_2(b(x_1)) \to f(q_2'(x_1), q_2''(x_1))$

Naive Construction of N

$(q_1, q_2)(a(x_1))$

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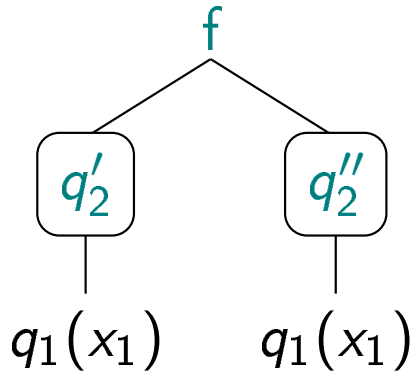

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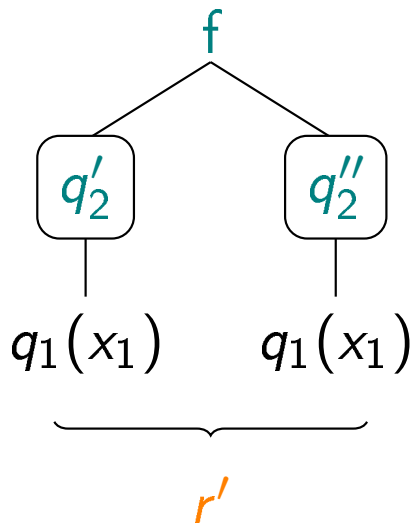
$q_2(b(q_1(x_1))) \Rightarrow^* r'$



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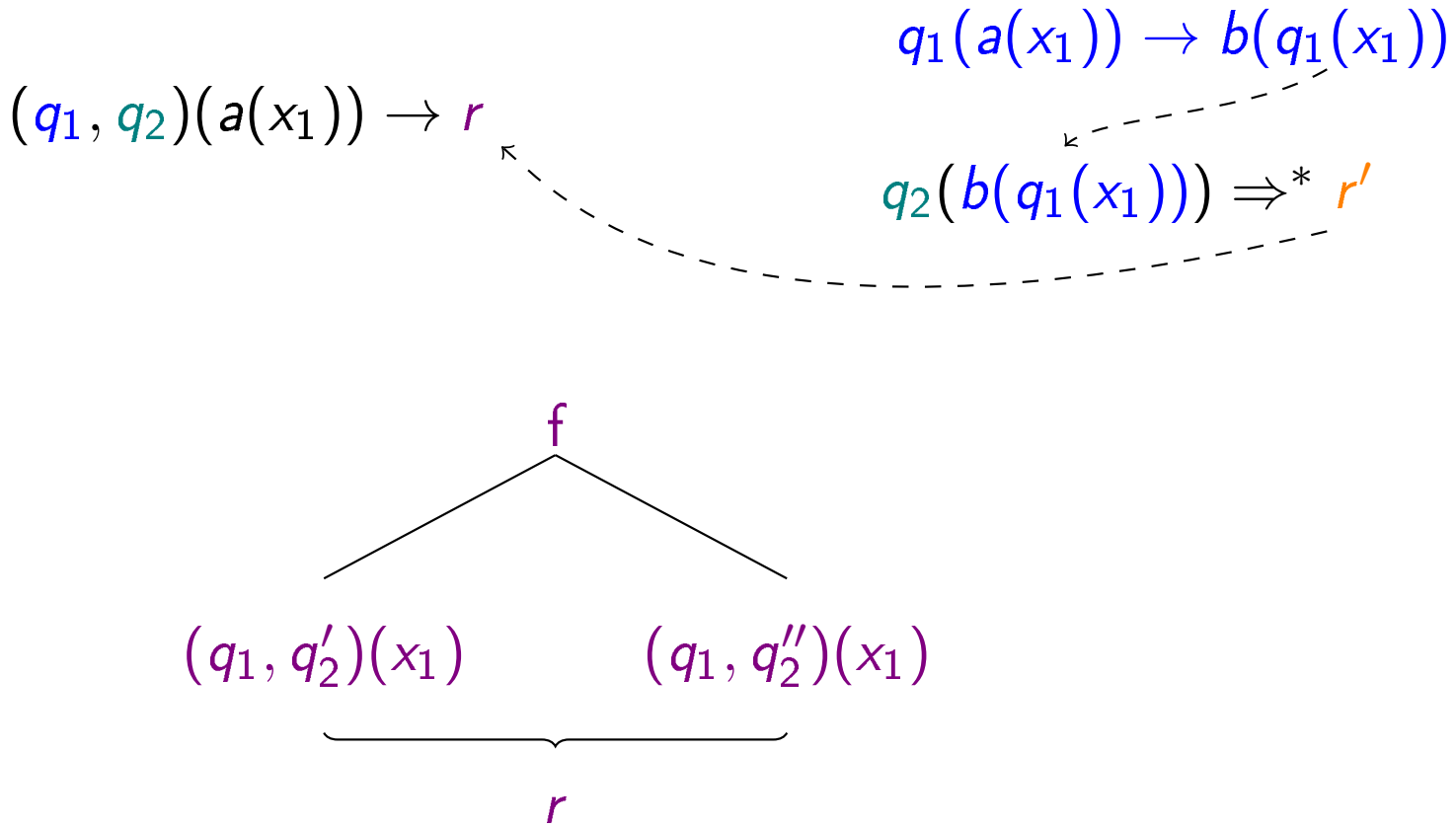

Naive Construction of N

Transducer N:

$$
\begin{array}{lcl}
(q_1, q_2)(a(x_1)) & \to & f\ ((q_1, q_2')(x_1), (q_1, q_2'')(x_1)) \\
(q_1, q_2'')(e) & \to & e
\end{array}
\qquad
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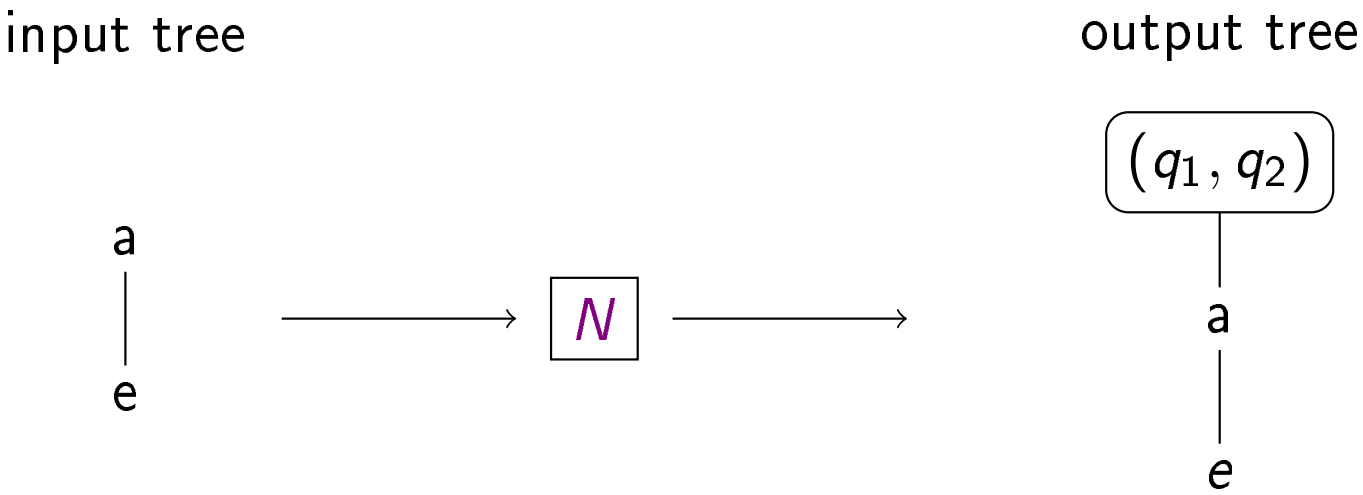
Transduction:



Naive Construction of N

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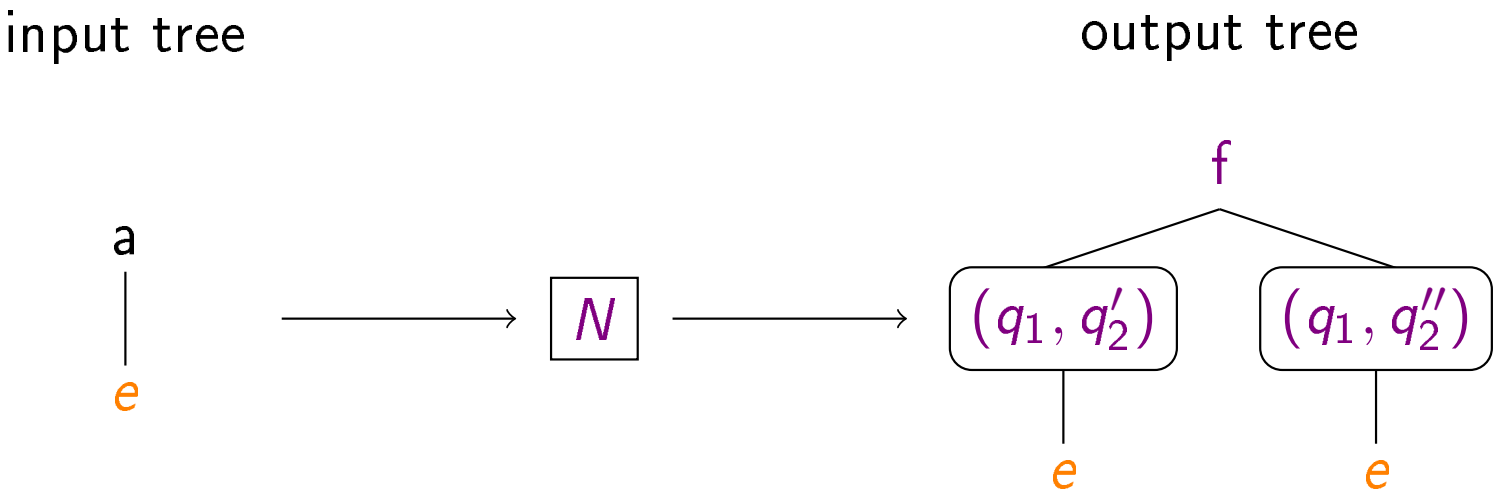

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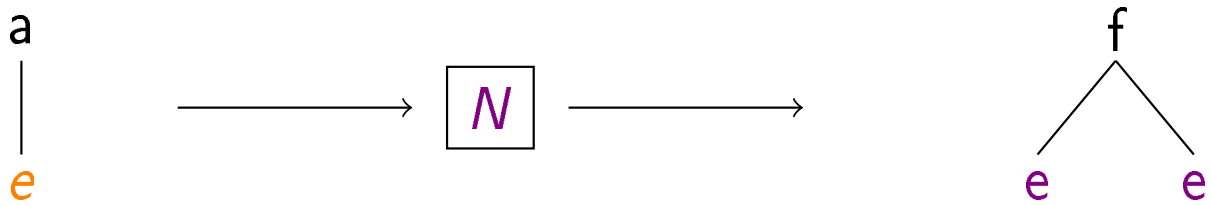
output tree



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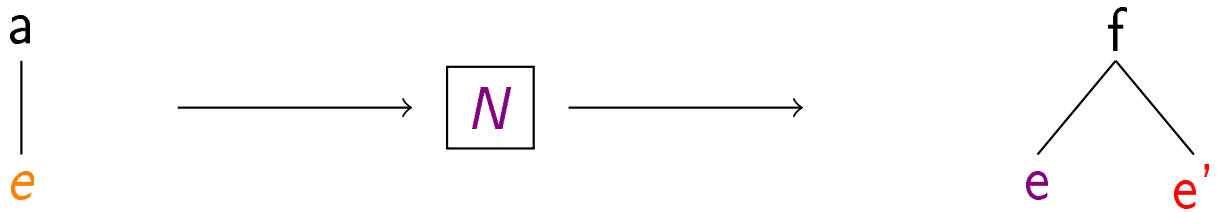
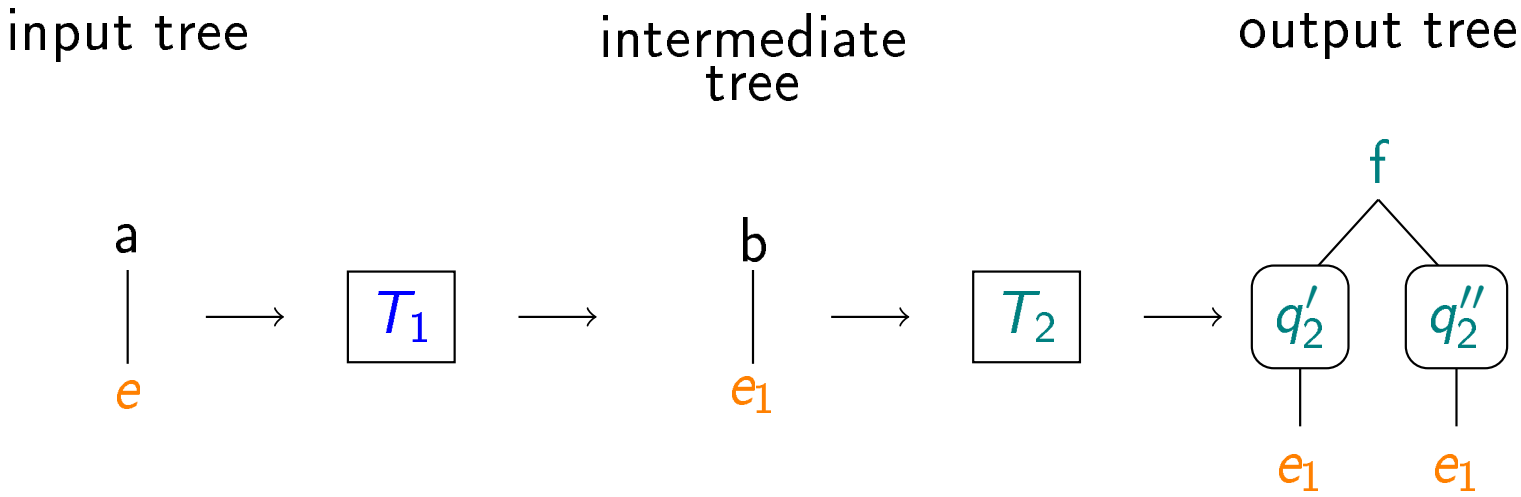


Problems of the naive Construction



q_2' and q_2'' process the tree e_1 produced by q_1 on input e

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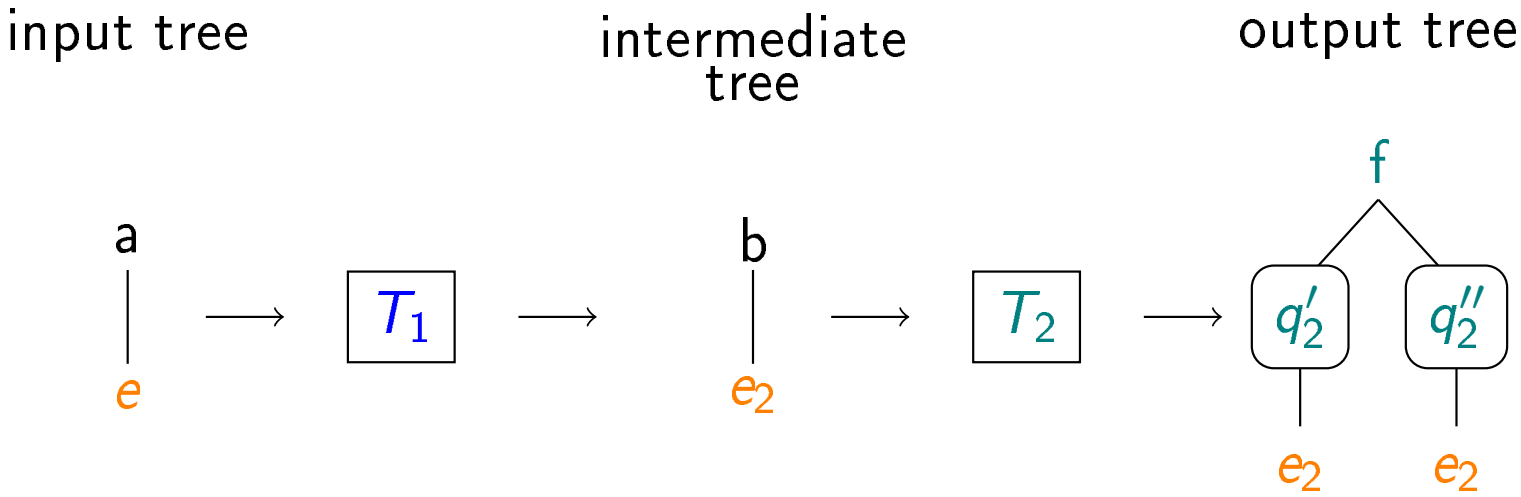

q_2' and q_2'' process the tree e_2 produced by q_1 on input e

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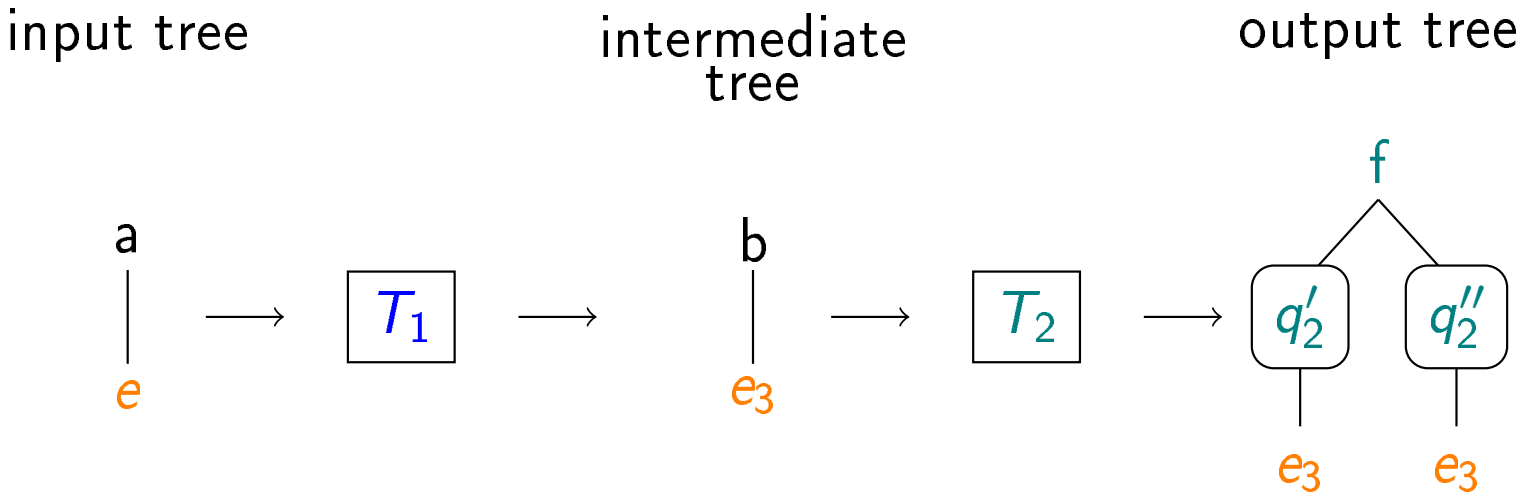
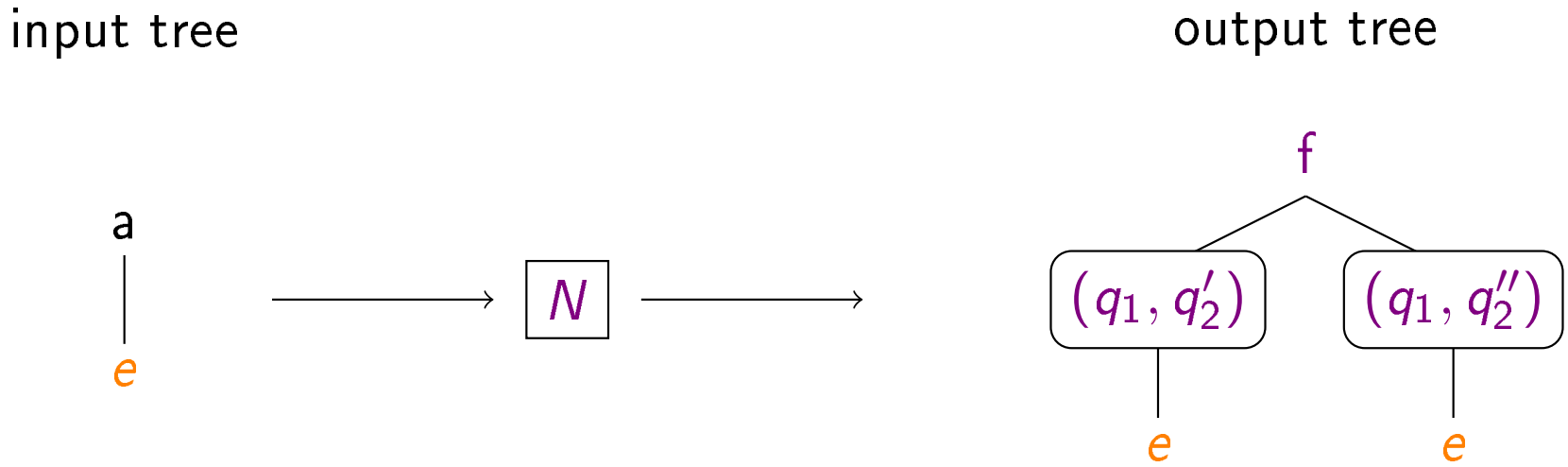


q_2' cannot process the tree e_3 produced by q_1 on input e

Problems of the naive Construction



$$(q_1, q_2')(e) \to e \begin{cases} q_1(e) \to e_1 \\ q_2'(e_1) \to e \end{cases} \qquad (q_1, q_2'')(e) \to e \begin{cases} q_1(e) \to e_1 \\ q_2''(e_1) \to e \end{cases}$$

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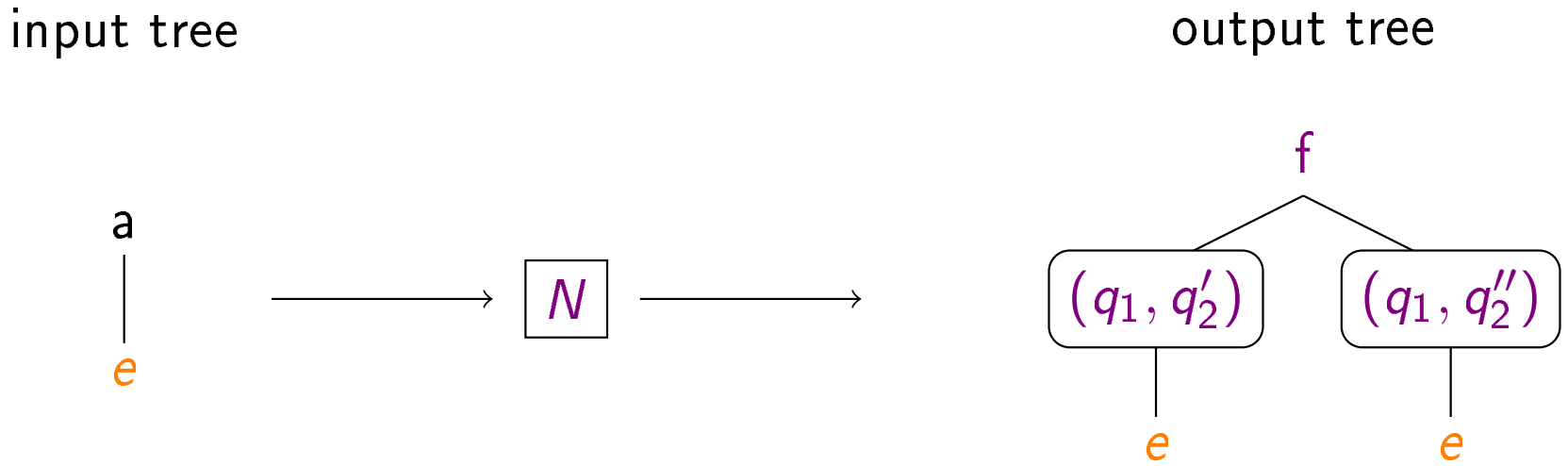

unfortunately, synchronization is not possible...

Observation

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Observation: It is sufficient that

1. (q_1, q_2') only guesses trees in $\mathrm{dom}(q_2') \cap \mathrm{dom}(q_2'')$
2. (q_1, q_2'') only guesses trees in $\mathrm{dom}(q_2') \cap \mathrm{dom}(q_2'')$

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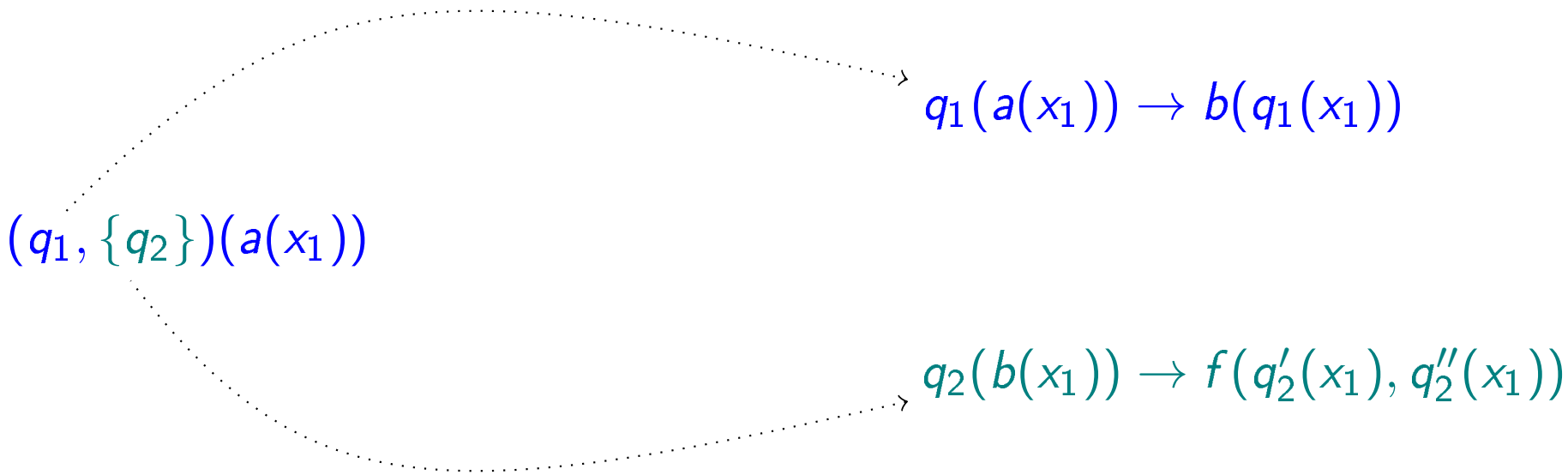
idea: T_1 simulates T_2 on its output



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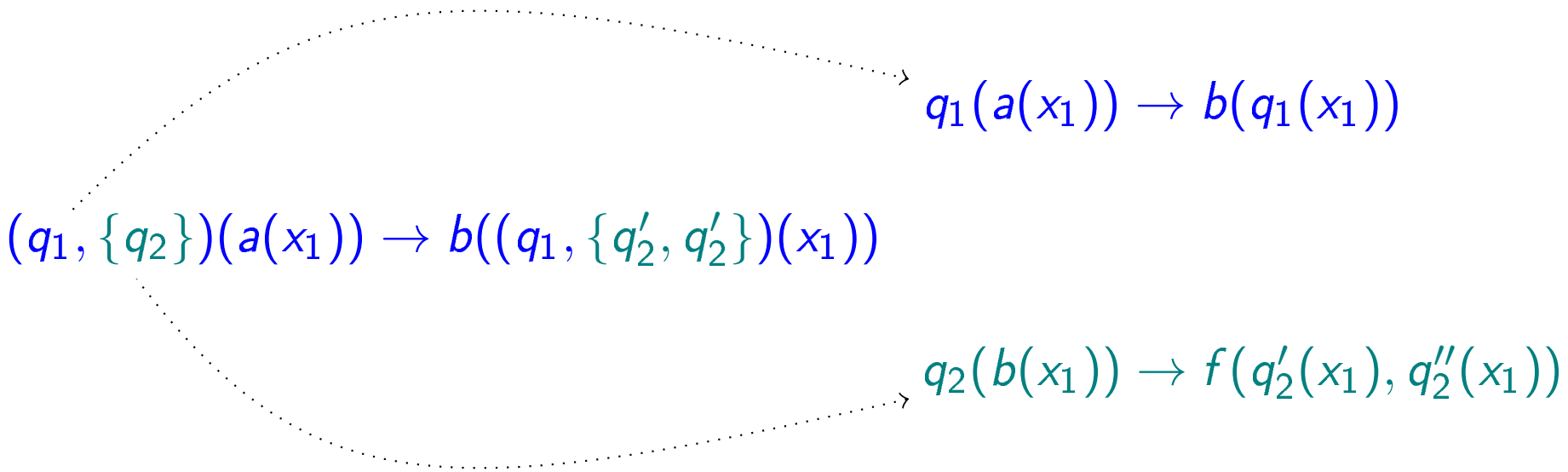
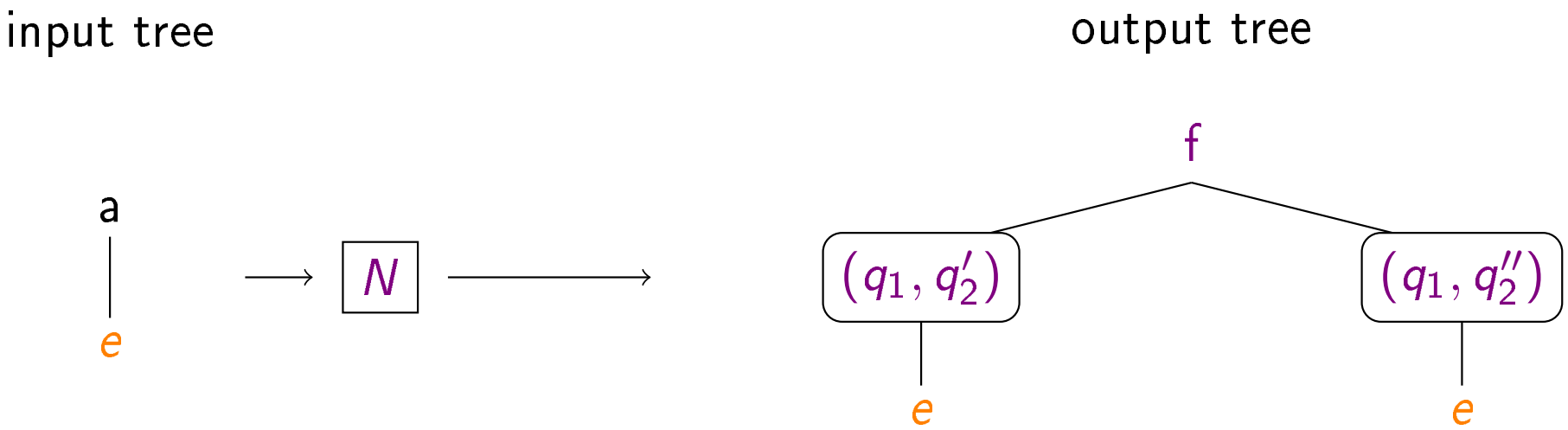


Idea:

Naive construction with the modified T_1:



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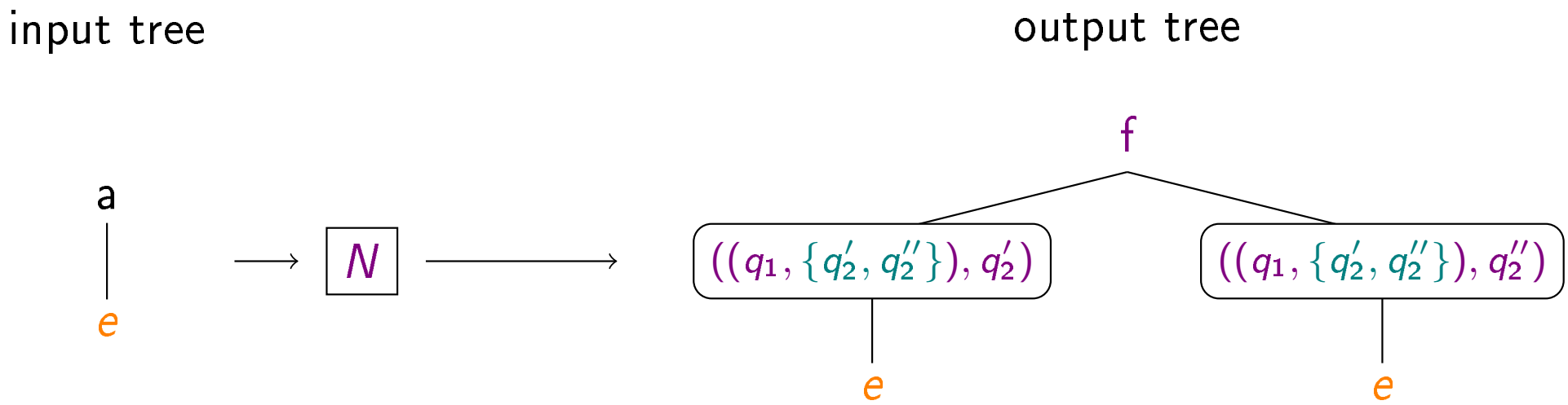

Deleting Rules

Transducer T_1:

$$\begin{array}{lcl} q_1(a(x_1, x_2)) & \to & b(q_1'(x_1), q_1''(x_2), q_1'''(x_2)) \\ q_1'(e) & \to & e \end{array}$$

$\mathrm{dom}(q_1')$ consists of trees whose leftmost leaf is e
$\mathrm{dom}(q_1'')$ consists of trees whose leftmost leaf is c

Transducer T_2:

$$\begin{array}{lcll} q_2(b(x_1, x_2, x_3)) & \to & q_2(x_1) & \\ q_2(e) & \to & e_j & \mid j = 1, 2. \end{array}$$

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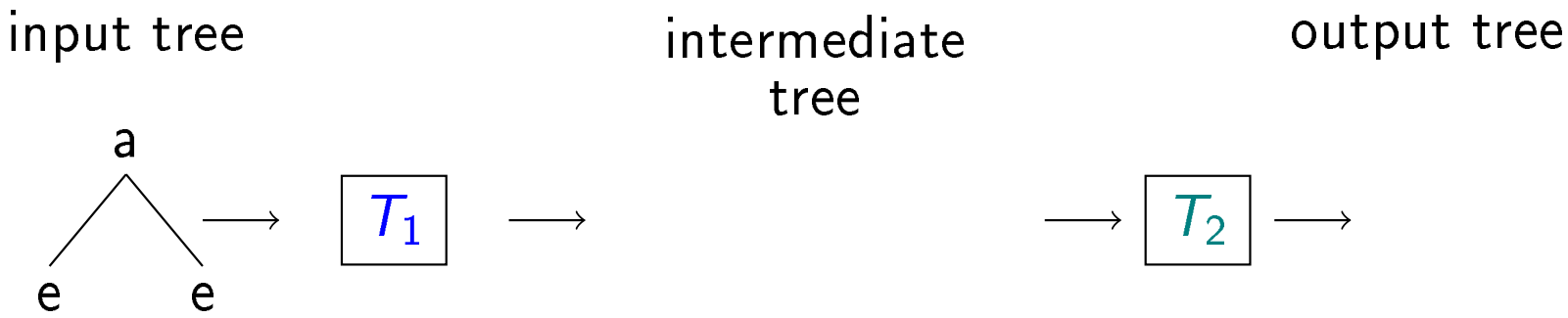

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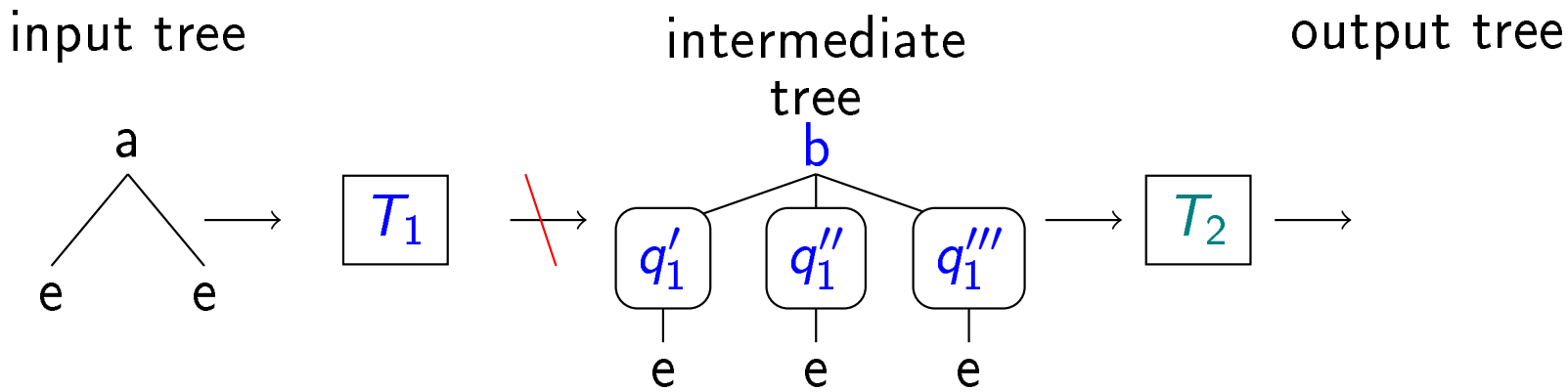

Naive Construction:

Transducer N:

$$(q_1, q_2)(a(x_1, x_2)) \rightarrow (q_1', q_2)(x_1) \qquad (q_1', q_2)(e) \rightarrow e_j \mid j = 1, 2$$

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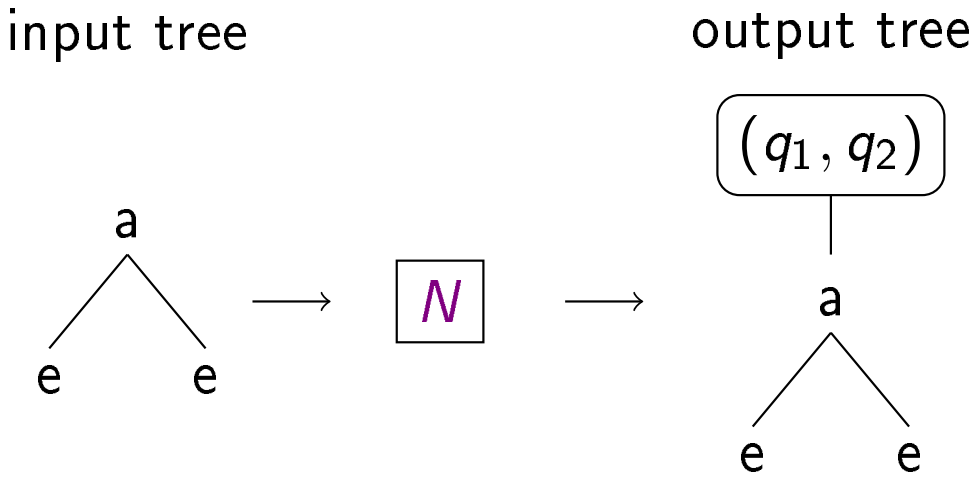

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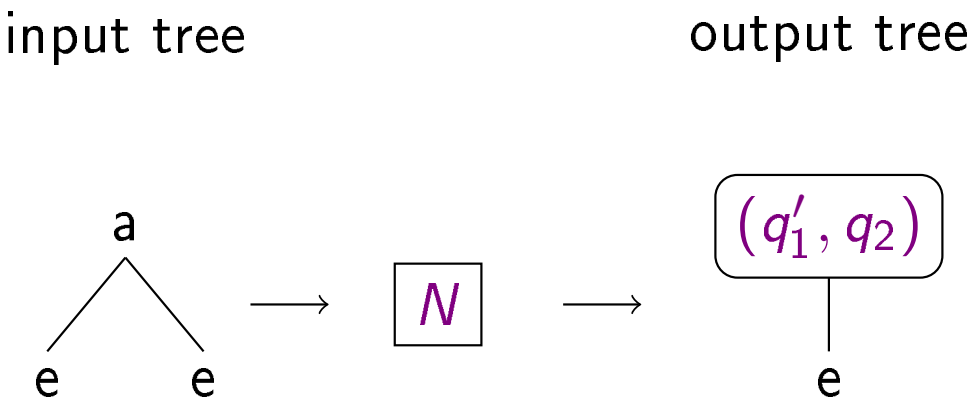

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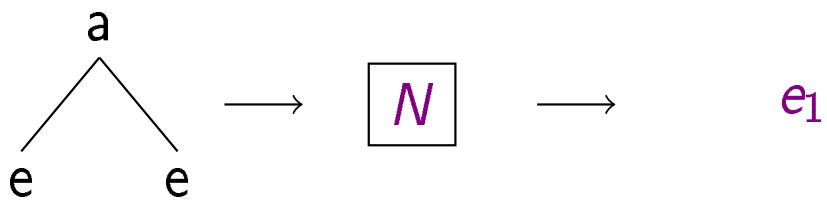
input tree output tree



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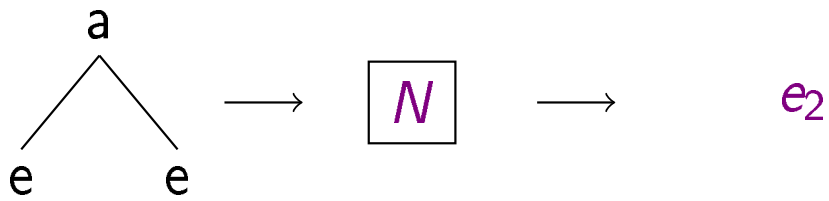

Idea:

Before processing a tree $a(s_1, s_2)$ with th rule

$$(q_1, q_2)(a(x_1, x_2)) \to (q_1', q_2)(x_1) \begin{cases} q_1(a(x_1, x_2)) \to b(q_1'(x_1), q_1''(x_2), q_1'''(x_2)) \\ q_2(b(x_1, x_2, x_3)) \to q_2(x_1) \end{cases}$$

test whether $s_1 \in \mathrm{dom}(q_1')$ and $s_2 \in \mathrm{dom}(q_1'') \cap \mathrm{dom}(q_1''')$ using Look-Ahead

Result

Combining our ideas for copying and deleting rules yields:

Theorem

Functionality for arbitrary compositions of top-down and bottom-up tree transducers is decidable. In the affirmative case, an equivalent deterministic top-down tree transducer with look-ahead can be constructed.

Thank you for your Attention