Designated-Verifier Linkable Ring Signatures with unconditional anonymity

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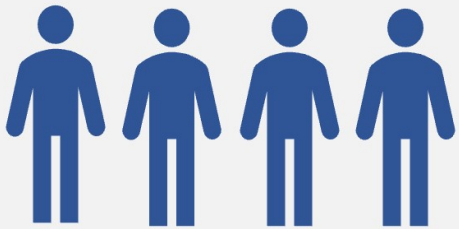
Overview

- anonymous feedback systems
- enhance anonymity of previously proposed constructions (DVLRS)
- prove the security properties in the $\mathcal{RO}$ model

An Anonymous Feedback System



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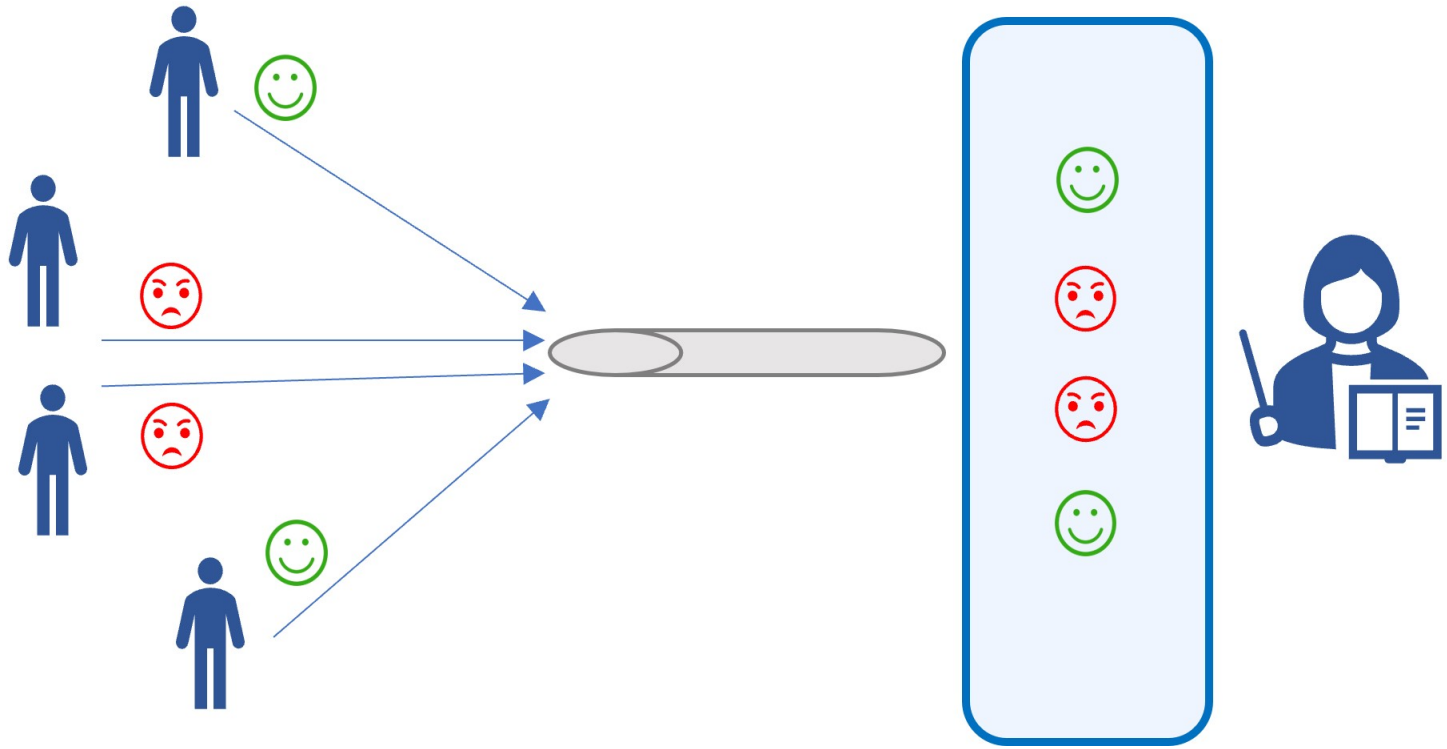

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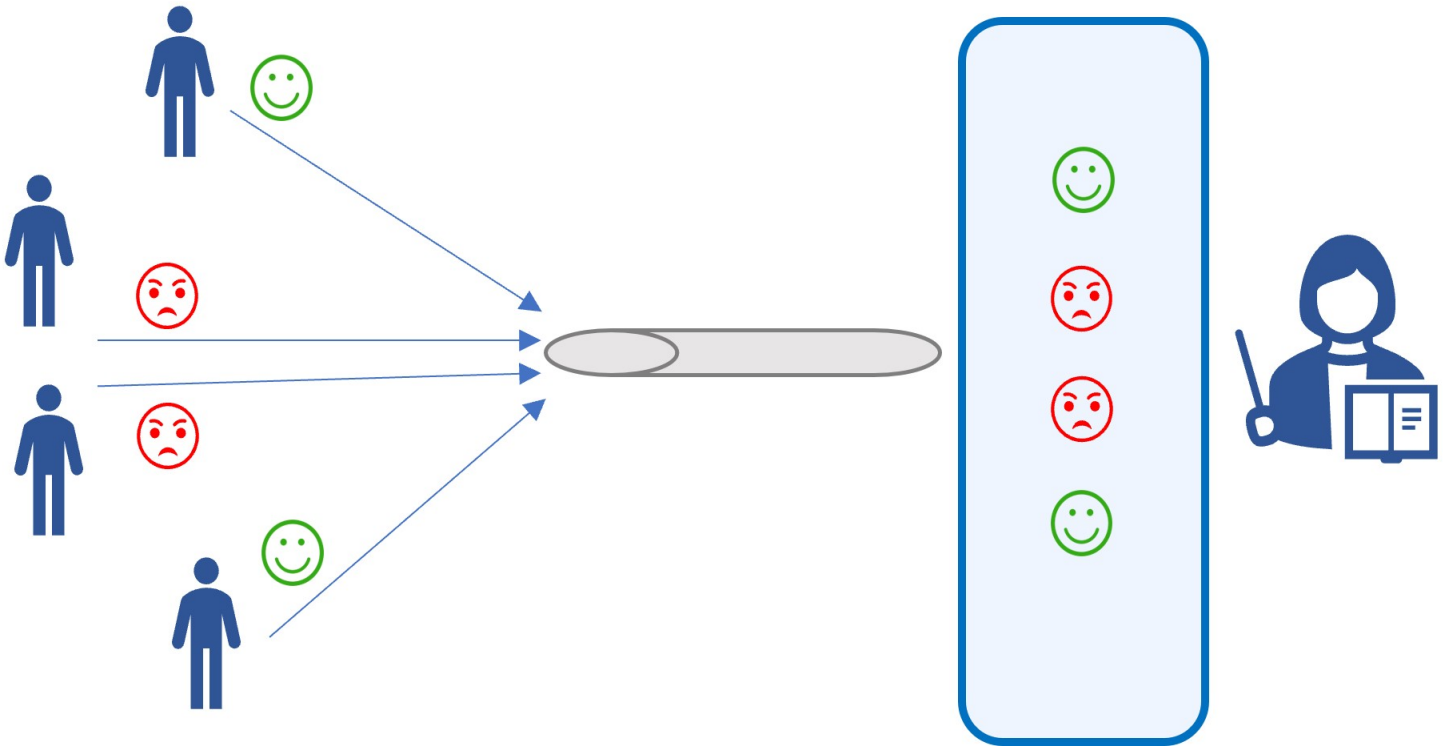

- Feedback from registered students
- Anonymous

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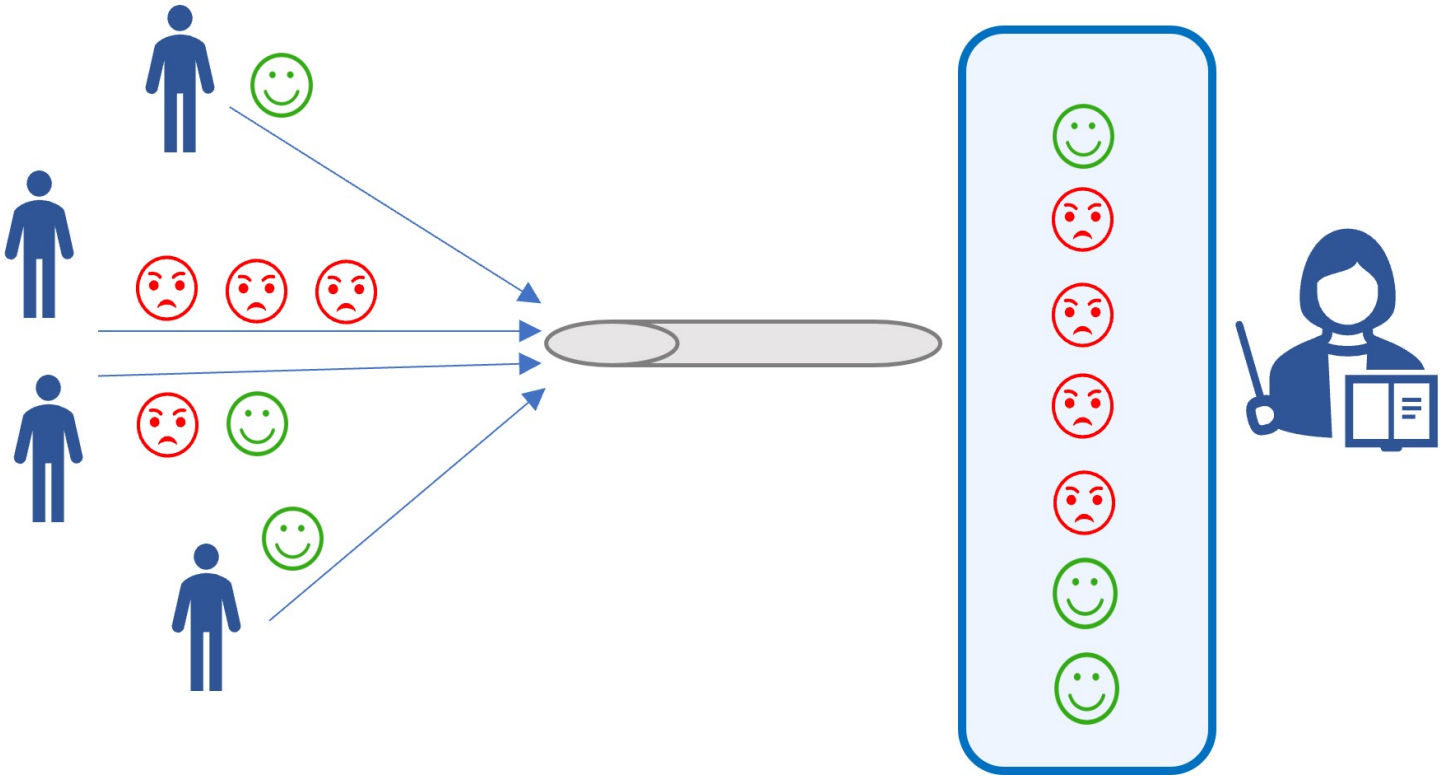

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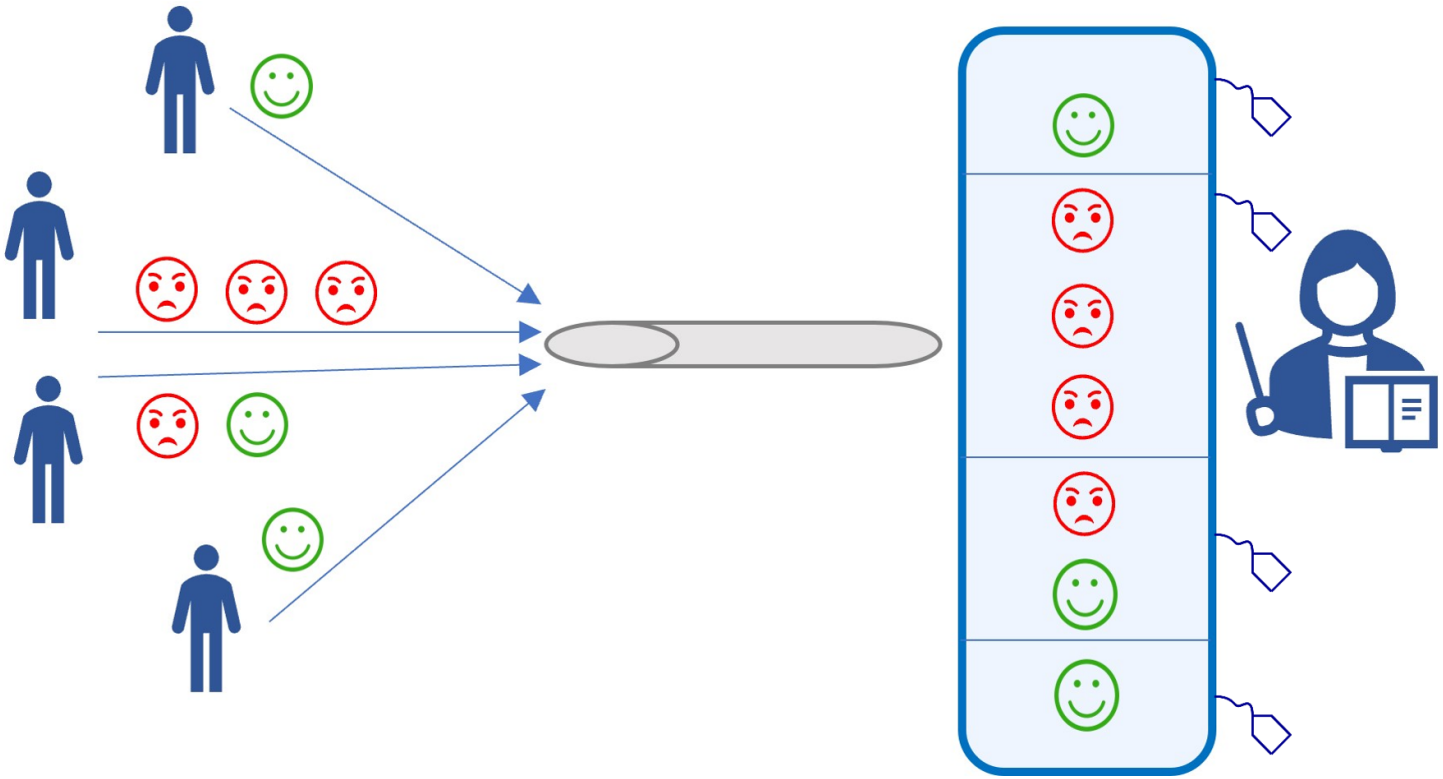

– The teacher can group feedback according to the linking tag, without knowing the actual identity of the student.

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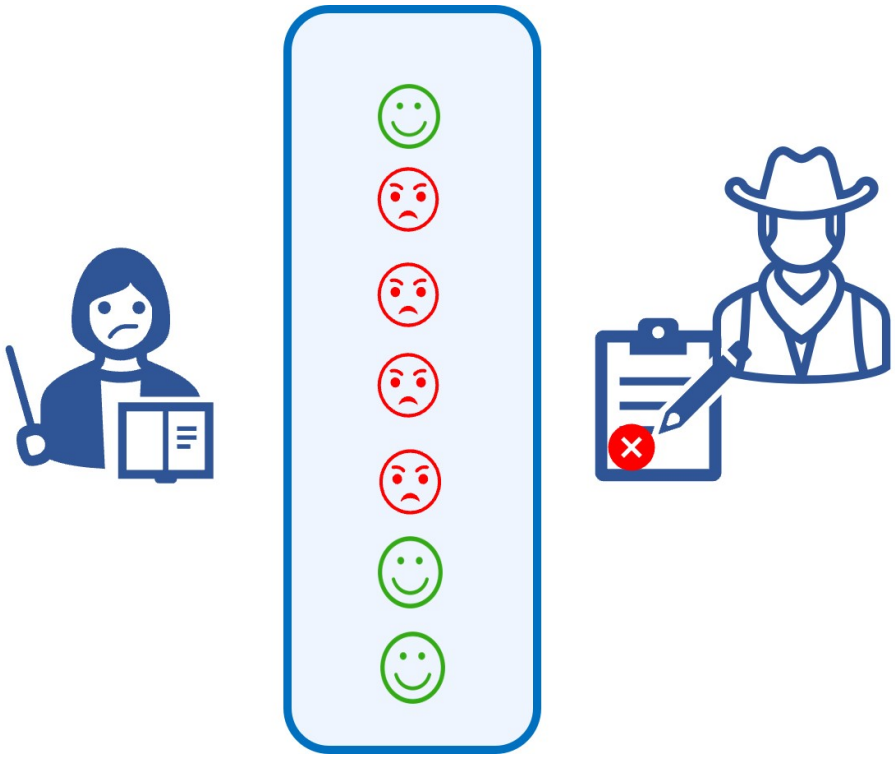


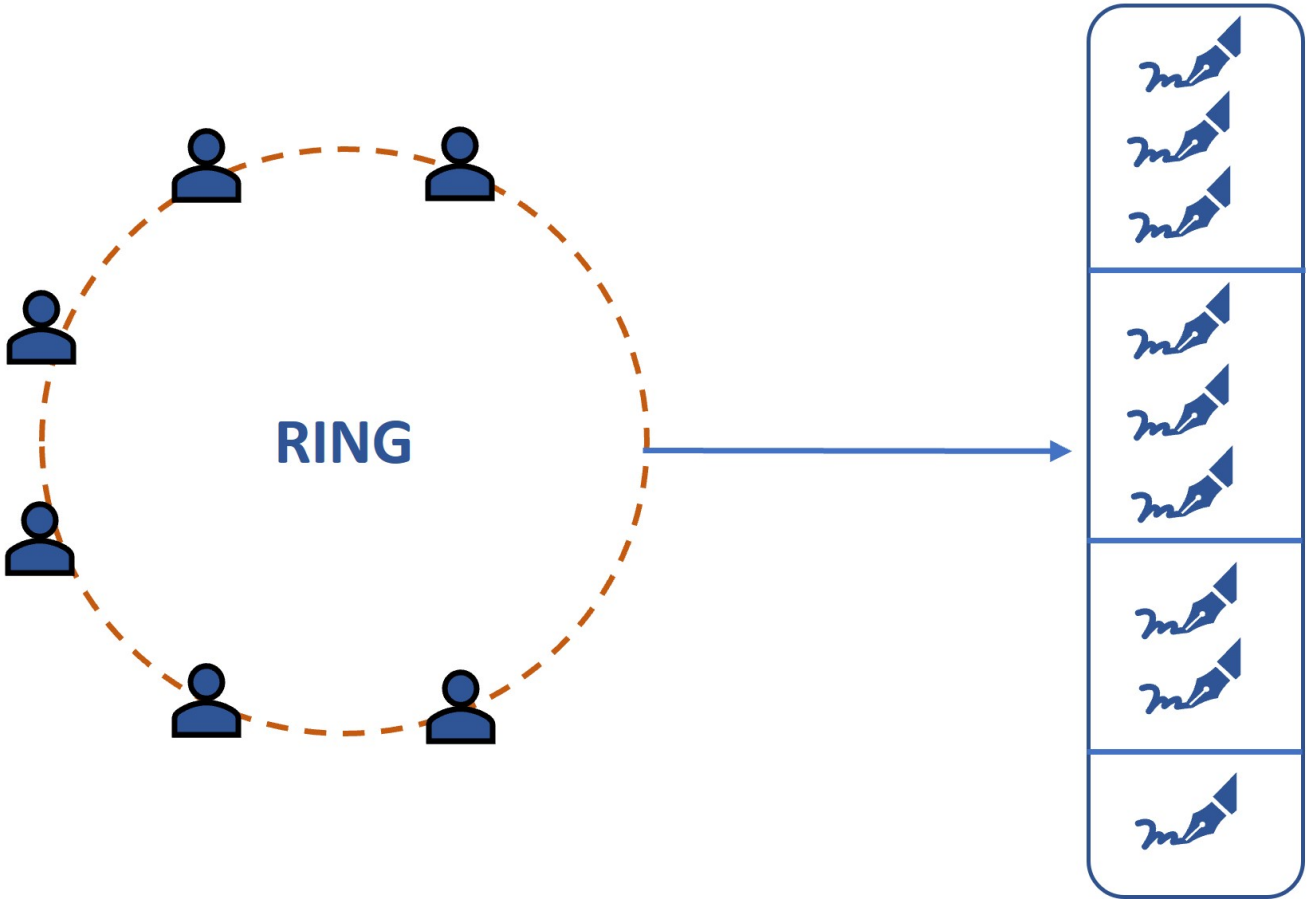
- Feedback only for the teacher
- Even if the teacher is forced to give away information

An Anonymous Feedback System

- A solution to the anonymous feedback system is given by DVLRS [Beh+21].
- DVLRS combines linkable ring signatures and designated verifier signatures.

[Beh+21] Behrouz, Grontas, Konstantakatos, Pagourtzis, Spyrakou. Designated Verifier Linkable Ring Signatures

Linkable Ring Signatures

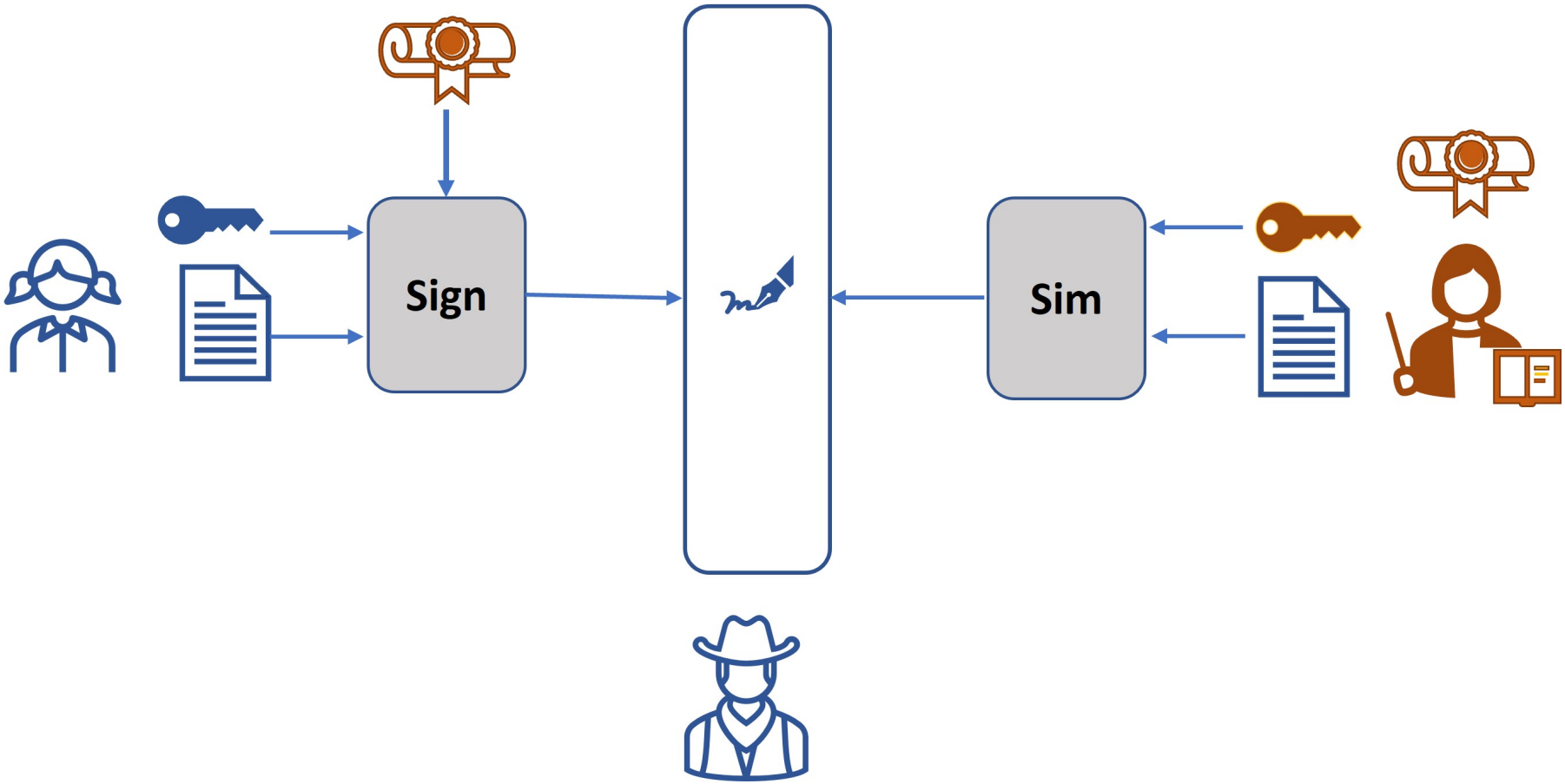


- Ring Signatures [RST01]
- Linkable Ring Signatures [LWW04]

[RST01] Rivest, Shamir, Tauman. How to Leak a Secret

[LWW04] Liu, Wei, Wong. Linkable Spontaneous Anonymous Group Signature for Ad Hoc Groups

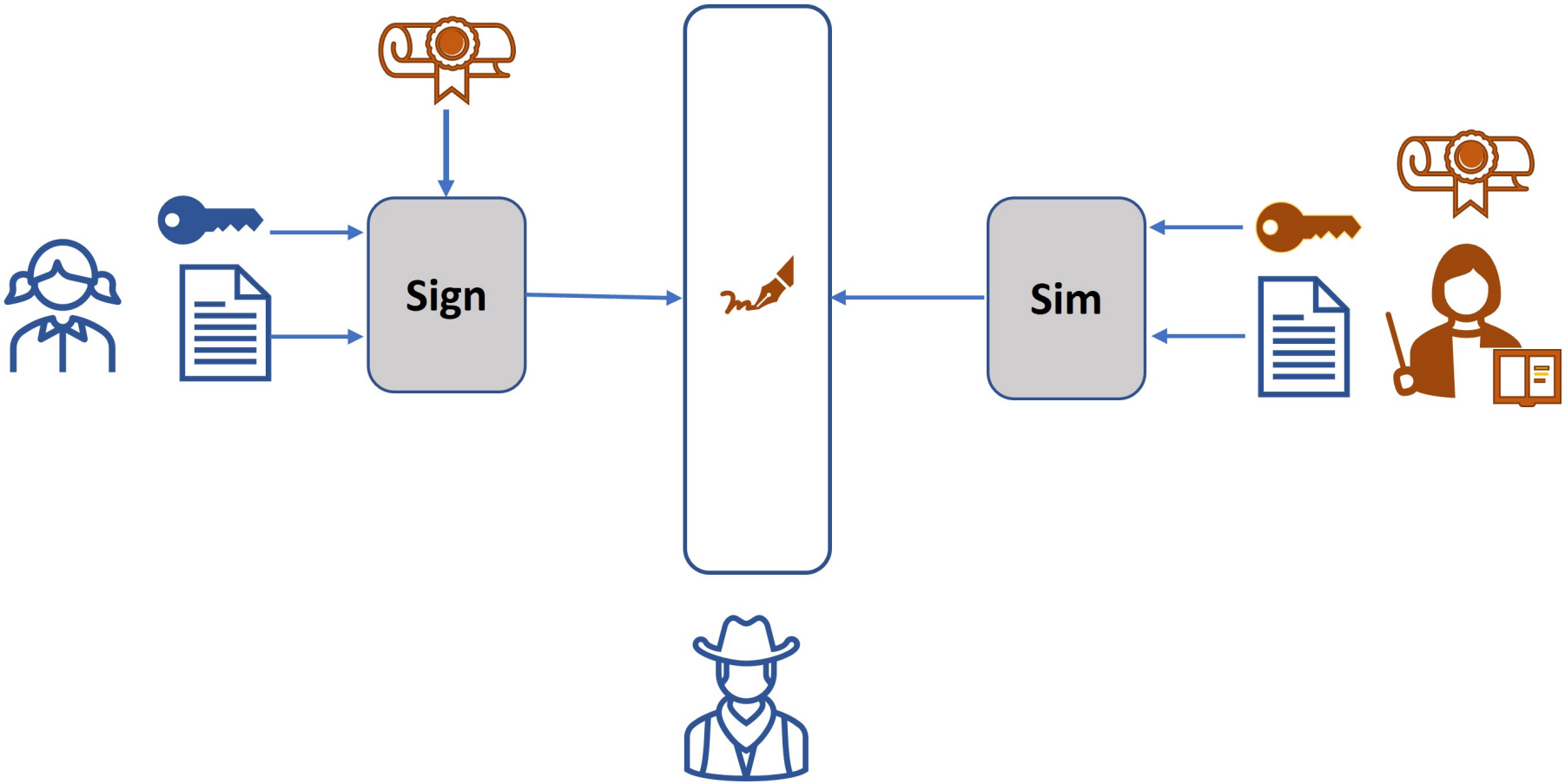
Designated-Verifier Signatures



- (Strong) Designated Verifier Signatures [JSI96]

[JSI96] Jakobsson, Sako, Impagliazzo. Designated Verifier Proofs and Their Applications

Designated-Verifier Signatures



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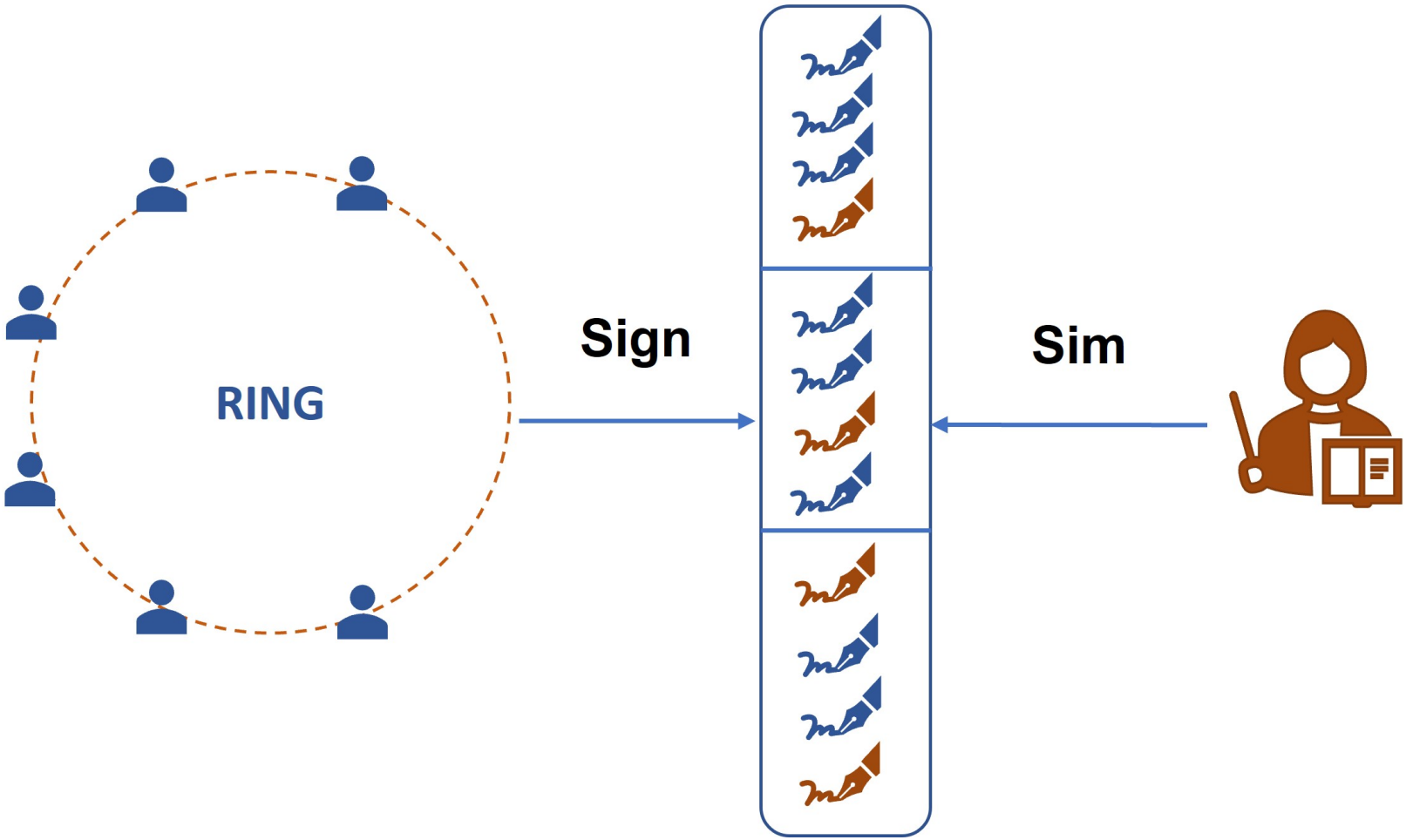
DVLRS Model



UDVLRS

- UDVLRS combines DVLRS [Beh+21] and ULRS [Liu+14]
- UDVLRS enhances the **anonymity** of DVLRS by making it unconditional.

[Beh+21] Behrouz, Grontas, Konstantakakos, Pagourtzis, Spyrakou. Designated Verifier Linkable Ring Signatures

[Liu+14] Liu, Au, Susilo, Zhou. Linkable Ring Signatures with Unconditional Anonymity.

UDVLRS Syntax

- $\mathsf{params} \leftarrow \mathsf{Setup}(\lambda)$
- $(\mathsf{sk}, \mathsf{pk}) \leftarrow \mathsf{KGen}()$
- $\sigma \leftarrow \mathsf{Sign}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \mathsf{sk}_\pi)$
- $\mathsf{pid} \leftarrow \mathsf{Extract}(\sigma)$
- $\sigma \leftarrow \mathsf{Sim}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \mathsf{sk}_D, \mathsf{pid})$
- $\{0,1\} \leftarrow \mathsf{Vrfy}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \sigma)$
- $\{0,1\} \leftarrow \mathsf{Link}(\sigma_1, \mathsf{ev}_1, \sigma_2, \mathsf{ev}_2)$

Adversarial Capabilities

Strong adaptive adversary

- Can add users to the system
- Can corrupt (take full control of) users
- Can request Signatures and Simulations

Security Properties

Unforgeability:
No one should be able to produce a valid signature except a member of the ring and the designated-verifier.

Anonymity:
No one, including the designated-verifier, should be able to identify the signer of a signature.
(Computationally unbounded $\mathcal{A}$)

Linkability:
If two signatures are signed with the same private key, they must be linked.

Non-slanderability:
No one should be able to maliciously link a signature to a specific ring member.

Non-transferability:
Signatures and simulations are indistinguishable.
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Construction

- $\mathbb{G}$, ord($\mathbb{G}$)$= q$, DLOG is hard.
- g, h generators of $\mathbb{G}$.
- Private keys $(x_i, y_i) \in \mathbb{Z}_q^2$, $i \in [n_L]$.
- Public keys $Z_i = g^{x_i} h^{y_i} \in \mathbb{G}$, $i \in [n_L]$.
- $\mathsf{H}_{\mathbb{G}} : \{0,1\}^* \to \mathbb{G}$, $\mathsf{H}_q\{0,1\}^* \to \mathbb{Z}_q$.

$\mathsf{Sign}(\mathsf{ev}, L, m, \mathsf{pk}_D, \mathsf{sk}_\pi)$

Signer π **computes**:

- $e \leftarrow \mathsf{H}_G(\mathsf{ev})$
- t : linking tag
- K: uses the public keys of the ring
- K': uses the linking tag t
- K'': uses the DV's pk

- $t \leftarrow e^{x_\pi}$
- $K \leftarrow g^{r_x} h^{r_y} \cdot \prod_{\substack{i \in [n] \\ i \neq \pi}} Z_i{}^{c_i + w_i}$
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Signer π **commits** on:

$c \leftarrow \mathsf{H}_q(m, L, \mathsf{ev}, t, K, K', K'')$

$$c_\pi \leftarrow c - \sum_{\substack{i=1, \\ i \neq \pi}}^{n} c_i$$
$$\tilde{x} \leftarrow (r_x - (c_\pi + w_\pi) x_\pi) \mod q$$
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Signer π **computes**: $c_\pi, \tilde{x}, \tilde{y}$

$$\sigma = (t, \tilde{x}, \tilde{y}, r, s, c_1, \ldots, c_\pi, \ldots, c_n, \{w_i\}_{i=1}^{n})$$

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DV **chooses**: t linking tag

DV **computes**:

- $e \leftarrow \mathsf{H}_G(\mathsf{ev})$
- K_D: uses the public keys of the ring
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- K''_D: commit on random values

- $K_D \leftarrow g^{\chi} h^{\psi} \cdot {Z_1}^{\alpha} \cdot \prod_{i=2}^{n} {Z_i}^{c_i + w_i}$
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- $K_D \leftarrow g^{\chi} h^{\psi} \cdot {Z_1}^{\alpha} \cdot \prod_{i=2}^{n} {Z_i}^{c_i + w_i}$
- $K'_D \leftarrow e^{\chi} \cdot t^{\alpha + \sum_{i=2}^{n} c_i + w_i}$
- $K''_D \leftarrow g^{\beta} h^{\gamma} \cdot \prod_{i=2}^{n} g^{w_i}$

$$c_1 \leftarrow c - \sum_{i=2}^{n} c_i$$
$$w_1 \leftarrow \alpha - c_1 \mod q$$
$$r \leftarrow (\beta - w_1) \cdot {x_D}^{-1} \mod q$$
$$s \leftarrow \gamma - r \cdot y_D \mod q$$

$$\sigma = (t, \tilde{x}, \tilde{y}, r, s, c_1, \{c_i\}_{i=2}^{n}, w_1, \{w_i\}_{i=2}^{n})$$

$\mathsf{Vrfy}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \sigma)$

$$\sigma = (t, \tilde{x}, \tilde{y}, r, s, \{c_i\}_{i=1}^n, \{w_i\}_{i=1}^n)$$

compute:

- $e \leftarrow \mathsf{H}_G(\mathsf{ev})$
- K: PoK of one of the secret keys of the ring
- K': proof of using the correct linking tag
- K'': PoK of the DV's secret key

- $K \leftarrow g^{\tilde{x}} h^{\tilde{y}} \cdot \prod_{i=1}^{n} Z_i{}^{c_i + w_i}$
- $K' \leftarrow e^{\tilde{x}} \cdot t^{+\sum_{i=1}^{n} c_i + w_i}$
- $K'' \leftarrow h^s \cdot \mathsf{pk}_D^r \cdot \prod_{i=2}^{n} g^{w_i}$

Verify that:

$$\sum_{i=1}^{n} c_i = \mathsf{H}_q(\mathtt{m}, L, \mathsf{ev}, t, K, K', K'')$$

$\mathsf{Vrfy}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \sigma)$

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$\mathsf{Vrfy}(\mathsf{ev}, L, \mathtt{m}, \mathsf{pk}_D, \sigma)$

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Verify that:

$$\sum_{i=1}^{n} c_i = \mathsf{H}_q(\mathtt{m}, L, \mathsf{ev}, t, K, K', K'')$$

Security Analysis (Proof Sketch)

Unforgeability

- Assume adversary $\mathcal{A}$ can forge with non negligible probability
- Use forking lemma
- Solve the MDLR problem
- The MDLR problem:
  Let $\mathbb{G}$ be a cyclic group of prime order q, $\mathbb{G} =< g >$ and $Y_1, Y_2, \ldots, Y_n \leftarrow \mathbb{G}, Y_1 \neq 1_{\mathbb{G}}$. A solution to the MDLR problem is a tuple $(\phi_1, \phi_2, \ldots, \phi_n) \in \mathbb{Z}_q^n$ such that $Y_1 \cdot Y_2^{\phi_2} \cdots Y_n^{\phi_n} = g^{\phi_1}$ and $\sum_{i=1}^{n} \phi_i \neq 0 \pmod{q}$
- MDLR is equivalent to DLOG, which yields contradiction

Security Analysis (Proof Sketch)

Unforgeability

Anonymity

- Let σ be a signature and t its linking tag.
- Note that only the linking tag can leak out information about the identity of the signer.
- Assume $\mathcal{A}$ can solve DLOG and find x such that $e^x = t$
- $\mathcal{A}$ cannot tell to which public key the linking tag t corresponds

Security Analysis (Proof Sketch)

Unforgeability

Anonymity

Linkability

- Assume adversary $\mathcal{A}$ owns a pair of secret key in the ring L and can produce 2 pairwise unlinkable signatures
- Use forking lemma
- Solve the MDLR problem, which yields contradiction

Security Analysis (Proof Sketch)

Unforgeability

Anonymity

Linkability

Non-slanderability

- Assume adversary $\mathcal{A}$ is given a signature σ and can produce a signature σ' s.t. $\mathsf{Link}(\sigma) = \mathsf{Link}(\sigma')$
- Use forking lemma
- Solve the DLOG problem, which yields contradiction.

Security Analysis (Proof Sketch)

Unforgeability

Anonymity

Linkability

Non-slanderability

Non-transferability

- Given a $\sigma \leftarrow$ Sign and a $\sigma' \leftarrow$ Sim on the same $\mathtt{ev}, L, DV, t$ the components of σ, σ' follow the same distributions.

Comparison of Ring Signature Schemes

Signature	Unforgeability	Anonymity	Linkability	Non-Slanderability	Non-Transferability
RS[AOS02]	DLOG	Unconditional	–	–	–
LRS [LWW04]	DLOG	DDH	DLOG	DLOG	–
ULRS [Liu+14]	DLOG	Unconditional	MDLR	DLOG	–
DVLRS [Beh+21]	DLOG	DDH	DLOG	DLOG	Unconditional
UDVLRS	MDLR/DLOG	Unconditional	MDLR	DLOG	Unconditional

- DVLRS inherits the anonymity of LRS, that is weaker than RS
- UDVLRS achieves unconditional anonymity, inherited by ULRS

[AOS02] Abe, Ohkubo, Suzuki. 1-out-of-n Signatures from a Variety of Keys.

[LWW04] Liu, Wei, Wong. Linkable Spontaneous Anonymous Group Signature for Ad Hoc Groups

[Liu+14] Liu, Au, Susilo, Zhou. Linkable Ring Signatures with Unconditional Anonymity.

[Beh+21] Behrouz, Grontas, Konstantakakos, Pagourtzis, Spyrakou. Designated Verifier Linkable Ring Signatures


Thank
you